“Financial Innovation,
Coordination Games, and Networks”

Dissertation
for the Faculty of Economics, Business Administration and Information Technology of the University of Zurich

to achieve the title of
Doctor of Economics

presented by

Miret Padovani
from Milan, Italy

approved at the request of

Prof. Dr. Paolo Vanini
Prof. Hyun Song Shin, Ph.D.

Miret Padovani

Route de Vandoeuvres 112, 1253 Vandoeuvres, Switzerland
Tel: +41 (0)79 379 21 60 Email: miretpadovani@yahoo.it

Work experience

Postdoctoral researcher (100% employment) **2010 - present**

- Geneva Finance Research Institute, University of Geneva, Switzerland.
- Position sponsored by the Geneva Financial Center.

Financial engineer (60-80% employment) **2008 - 2010**

- Portfolio Solutions team, Zürcher Kantonalbank, Switzerland.
- Software development for investment portfolio analysis and optimization. Extensive Java programming.
- Business cases for a variety of financial products: sustainable investment funds; sustainable real estate funds; microfinance funds and investment certificates; minimum variance investment certificates.
- In charge of the in-house continuing education in finance („Ausbildung Handel & Kapitalmarkt"), including lecture script, exercises, and exams.

Teaching assistant (50% employment) **2007 - 2009**

- Chair of Professor Paolo Vanini, Swiss Banking Institute, University of Zurich, Switzerland.
- Lectures, exercises, and exams for Financial Engineering I & II.
- Supervision of diploma and master theses on financial engineering topics.

+ Several student jobs (interpreter, restaurant hostess, hotel receptionist).

Education

Doctoral studies in Finance **2010**

University of Zurich, Switzerland.
Advisors: Paolo Vanini and Hyun Song Shin.
Thesis title: Financial innovation, coordination games, and networks.
Research Web page at http://www.miret.eu.

Master of Advanced Studies in Finance *(magna cum laude)* **2006**

ETH Zurich & University of Zurich, Switzerland.
Specialization: Quantitative Finance and Risk Management.

Master of Science in Economics *(distinction)* **2005**

University of Bath, UK.
Specialization: Development Economics.

Laurea (MA equivalent) in Economics *(summa cum laude)* **2003**

Catholic University of Milan, Italy.
Specialization: Economic Theory.

Research papers

M. Padovani & P. Vanini, An intergenerational cross-country swap, Swiss Finance Institute Research Paper 09-17 (*submitted*).
M. Padovani, Solidarity group lending and global games, working paper, 2010.

M. Padovani, Value-chain financing and local interaction games, working paper, 2010.
M. Padovani, Pricing and hedging multi-asset equity derivatives, work in progress.

Language skills

Fluent in Italian, English, German, French, and Arabic.

Computer skills

Business software: Microsoft Word, Excel, PowerPoint, Outlook, FrontPage; LaTeX; LotusNotes.
Statistical / econometric software: MatLab; R; SPSS; LimDep; Eviews.
Programming language: Java.

Awards and scholarships

PhD scholarship, Swiss Banking Institute, 2006.
Scholarship for abroad studies, Catholic University of Milan, 2004.
Merit prize, Famiglia Fondazione Legnanese, 2003.

Extras

Textbook project:
M. Padovani & P. Vanini, Advanced financial engineering: Modeling and trading stochastic volatility and correlation, lecture notes for Financial Engineering II.

Co-organized the PhD Quant Finance Day 2008, Zurich, Switzerland.
Event Web page at http://www.quant-finance.eu/PhDQF2008.htm.

References

Hyun Song Shin
Professor of Economics, Princeton University.
hsshin@princeton.edu

Paolo Vanini
Assistant Professor of Finance, University of Zurich.
Head of Structured Products, Zürcher Kantonalbank.
paolo.vanini@zkb.ch

Angelo Ranaldo
Economic Advisor & Member of Senior Management, Swiss National Bank.
angelo.ranaldo@snb.ch

The Faculty of Economics, Business Administration and Information Technology of the University of Zurich hereby authorizes the printing of this Doctoral Thesis, without thereby giving any opinion on the views contained therein.

Zurich, 14. April 2010

the Dean: Prof. Dr. Josef Falkinger

Executive summary and acknowledgements

My doctoral thesis uses an interdisciplinary approach to tackle important issues in finance, game theory, and network theory. The first paper, "An intergenerational cross-country swap", co-authored with Paolo Vanini, addresses the issue of intergenerational and international sharing of longevity and growth risks. Current research on worldwide demographic changes highlights the importance of longevity risk for financial markets and the need to devise optimal hedging vehicles. We present a potential financial innovation between two countries at different stages of economic development and with different long-term challenges. This 30-year-long swap is structured in such a way to capture the different timing of needed funds of the two countries and the funding capabilities of each generation: the more developed economy requires funds in the future to cover expenses for its ageing population, while the developing economy needs funds today to pay for educational, technological, and other infrastructural services. To price the swap, we apply an exponential-utility-based pricing method and define an interval of prices allowing a contract to be agreed upon. We show how the bid-ask spread varies with respect to the governments' risk and time preferences.

In the second paper, "Solidarity group lending and global games", I analyse microfinance solidarity group lending within a global game framework. Group members invest independently in either a safe or a risky project. *Ex ante* observation and coordination on the choice of projects is not always possible. Though risky projects allow microentrepreneurs to extract risk premia, safe projects raise the possibility of success of the group loan, in which case group members benefit from access to business support services provided by the microfinance institution. I show how private beliefs concerning the strength of social cohesion within the microentrepreneurial community induces coordination on safe project choices.

Microfinance lending is also the topic of the final paper, "Value-chain financing and local interaction games", but here I add a network perspective to the picture. Value-chain financing is of great importance to the agricultural sector of most developing countries. Microfinance institutions may contribute to the strength of chain links by handing out loans. I view an agricultural value chain as a network, where links between nodes denote social and business relationships between chain actors. By studying internode relationships in terms of local interaction games, I address the issue of whether and how network structure and social cohesion may spread coordination on safe investment choices from a local level to the entire value chain.

I am deeply grateful to my advisors, Paolo Vanini and Hyun Song Shin, for their support, guidance, and endless patience throughout the writing of this thesis. I also thank my colleagues at the Zürcher Kantonalbank for interesting discussions and comments on my research.

AN INTERGENERATIONAL CROSS-COUNTRY SWAP

MIRET PADOVANI AND PAOLO VANINI

Abstract. This paper addresses the issue of intergenerational and international sharing of longevity and growth risks. Current research on worldwide demographic changes highlights the importance of longevity risk on financial markets and the need to devise optimal hedging vehicles. We present a potential financial innovation between two countries at different stages of economic development and with different long-term challenges. This 30-year-long swap is structured in such a way to capture the different timing of needed funds of the two countries and the funding capabilities of each generation: the more developed economy requires funds in the future to cover expenses for its ageing population, while the developing economy needs funds today to pay for educational, technological, and other infrastructural services. To price the swap, we apply an exponential-utility-based pricing method and define an interval of prices allowing a contract to be agreed upon. We show how the bid-ask spread varies with respect to the governments' risk and time preferences.

1. Introduction

Countries at different stages of economic development face different long-term challenges. On the one hand, take a country such as Switzerland, which faces the challenge of being able to provide for the pensions and long-term care of its ageing population without jeopardizing its economic competitiveness.[1] On the other hand, take Egypt, which faces the challenge of being able to attract sufficient funds and encourage new businesses to foster its economic development; Egypt is not constrained by longevity-related expenses, and higher longevity due to better health and living conditions may actually signify higher work productivity and faster economic growth. So Switzerland faces longevity risk, while Egypt faces growth risk. Rather than bear those risks, stakeholders in the two countries may benefit from transferring them to the financial markets. Yet the question is how.

Date: First version: April 2009. This version: February 18, 2010.

Key words and phrases. Financial innovation; longevity risk; old-age dependency ratio; growth risk; exponential-utility-based pricing; international finance; intergenerational risk-sharing; international risk-sharing.

We are thankful to Michel Habib, Hyun Song Shin, Alexander Wagner, and participants at the seventh Swiss doctoral workshop in finance (Gerzensee, June 2008) for helpful comments on a preliminary draft of the paper. Special thanks also go to our colleagues at the Zürcher Kantonalbank for interesting discussions and feedback. Part of this research has been carried out within the research project on "Credit risk and non-standard sources of risk in finance" led by Rajna Gibson at the National Centre of Competence in Research "Financial Valuation and Risk Management" (NCCR FINRISK). The NCCR FINRISK is a research instrument of the Swiss National Science Foundation.

[1] See the 2008 Swiss Health Observatory study [SBJR+08].

	2005	**2020**	**2030**	**2040**
Australia	AAA	AA	BBB	Non-IG
Canada	AAA	AAA	AAA	AA
France	AAA	A	Non-IG	Non-IG
Italy	AA	A	Non-IG	Non-IG
Japan	AA	Non-IG	Non-IG	Non-IG
Spain	AAA	AAA	BBB	Non-IG
Sweden	AAA	AAA	A	Non-IG
United Kingdom	AAA	AAA	A	Non-IG
USA	AAA	BBB	Non-IG	Non-IG

TABLE 1.1. Hypothetical projected long-term sovereign ratings according to baseline scenario if no adjustment in government budget occurs. *Source*: [Kra02].

We may very naively divide the world economy into two groups: developed and developing economies; or - political correctness aside - rich and poor. The former countries are short longevity, the latter are long longevity.

The contribution of this paper is threefold. We delineate the structure of a potential future financial innovation enabling one party - the rich country - to mitigate its longevity risk and the other - the poor country - to widen its pool of funding options. This intergenerational cross-country swap would mirror the different timing of needed funds of the two countries and the funding capacities of each generation. Secondly, we price the swap using an exponential-utility-based pricing method and determine the highest possible bid and lowest possible ask price. Finally, we show how the bid-ask spread varies with governments' risk and time preferences.

This swap can be seen as a "radical financial innovation", in the interpretation given by Shiller [Shi04b]. Indeed, this swap permits risk management to be extended far beyond its former realm, covering a new class of risk: population ageing. It also changes the assumptions about what can be insured and hedged - the old-age dependency ratio - and has a potential major impact on human welfare.

A crucial aspect in this framework is the interrelationship between longevity and growth risks, since inadequate management of the high costs associated with an ageing society may lead a country to economic deterioration. A 2002 Standard & Poor's study [Kra02] documents how ageing-related government liabilities may result in downgrades of sovereign ratings if no adjustment in government budget occurs (see table 1.1). This highlights the important role governments should take as managers of key long-term risks related to population ageing, as discussed by Groome *et al.* [GBH+06]. The literature investigating the economic effects of population ageing on financial markets and possible policy reforms is quite vast;[2] we refer to Groome *et al.* [GBR06] for a review of the most recent research to date.

Our interest here lies in international financial innovations motivated by worldwide asymmetric demographic trends. Bryant [Bry06] and Batini *et al.* [BCM06] independently study the interactions between developed and developing countries

[2]See, among others, Abel [Abe03] and Brooks [Bro02].

as their populations follow different evolution paths. Their analyses show that population ageing in industrialised countries will reduce growth and negatively affect savings and investments; on the other hand, developing countries will enjoy a "demographic dividend" that should result in stronger growth over the next couple of decades, before ageing sets in. Takling pretty much the same issue, Alho and Borgy [AB07] employ a multi-regional model to analyze the uncertainty induced into key macroeconomic variables by uncertain future demographics. The authors observe that the macroeconomic adjustments can differ substantially if they consider independence or high correlation across the regions.

In a series of thought-provoking studies, Robert Shiller and his co-authors stress the desirability of an international risk transfer of economic growth risks.[3] Consider the exemplifying "radical financial innovation" suggested by Shiller [Shi04a]:

> "Suppose that a ten-year contract were made between a poor country on one side - in this example, India - and such wealthier countries as Canada, Mexico, the United States, Brazil, Japan, France, Germany, Italy, and the United Kingdom on the other, to swap unexpected future changes in Indian GDP for unexpected future changes in the combined GDPs of the other countries. (Shiller [Shi04a])"

The swap we suggest differs from the one above in that it combines growth with longevity risk. Eijffinger and Wagner [EW03] show that international risk-sharing gains through existing financial assets actually result from intertemporal rather than cross-sectional gains, particularly because of less severe incentive problems. Such evidence provides strong support for the idea that a country which is short longevity would gain if it were to transfer its longevity risk through an intergenerational agreement with another country which is long longevity. The open question this paper aims to answer is: how can one concretely and successfully structure derivatives for a simultaneous intergenerational and international risk transfer?

Our derivative idea is also related to several recent studies in financial engineering - namely, on longevity-based securities,[4] economic derivatives,[5] as well as

[3] See Shiller [Shi04a] and Athanasoulis and Shiller [AS01].

[4] The idea behind derivatives such as longevity bonds (see Blake and Burrows [BB01], Blake *et al.* [BCDM06]) and longevity swaps (see Dowd *et al.* [DBCD06]) is a risk-exchange contract between two parties, one being short longevity and the other being long longevity. Typical counterparties for such a contract may be the sponsor of a pension plan and a life insurer. The payoffs of these derivatives depend upon the realized value of an index of longevity rates. Several investment banks and option exchanges - namely, JPMorgan, Goldman Sachs, Credit Suisse, and Deutsche Börse - have recently developed longevity indices with the aim to contribute to the take-off of the longevity derivatives market. Mitchell *et al.* [MPSY06] provide an interesting overview of financial product innovations in this field.

[5] Economic derivatives allow investors to take direct positions on the outcomes of macroeconomic data releases. The first economic derivatives - futures and options on non-farm payroll figures - started trading in 2002 on the Chicago Mercantile Exchange but were subsequently discontinued in 2007. Among the reasons given were lack of interest and illiquidity; indeed, little disagreement on the outcome of data makes it difficult for traders to find a counterparty willing to take the other side of the trade. There is ongoing work on how to best structure financial derivatives to mitigate event risks, such as adverse fluctuations in unemployment or national income figures (see Gadanecz *et al.* [GMU07]). Similar to the longevity derivatives market, the economic derivatives market is currently struggling to take off. Economic derivatives do entail, however, lower basis risk than more conventional instruments for taking positions on macroeconomic data.

commercial microfinance.[6] We do not discuss this literature here, but do highlight their importance, since they give an idea of the challenges encountered when engineering, pricing, and hedging innovative financial products.

The remainder of the paper is organized as follows. The next section illustrates the structuring of the swap and justifies the choice of the old-age dependency ratio as its underlying variable. The following section applies an exponential-utility-based method to price this innovative payoff in an incomplete market. Section 4 discusses how the arbitrage-free interval of possible swap prices varies with respect to the model parameters. Section 5 concludes and outlines future research work.

2. Structuring the swap

The financial innovation we present is an agreement between a rich country's government and a poor country's government to exchange a sequence of cash flows at specified settlement dates. Its structure is designed in such a way to capture the temporal asymmetry between the two countries: the richer country needs funds in the future when it will have to cover expenses for the elderly, while the poorer country needs funds today to pay for educational, technological, and other infrastructural services.

The swap has a long-term duration, say 30 years as for available government debt. The rich country represents the fixed leg, the poor country the floating leg. The net cash flows from rich to poor are much higher in the first years of the contract's life, but this asymmetry is reversed over time, mirroring the different timing of needed and available funds stated above.

The swap's structure begs the question of how far ahead one can forecast growth and whether 30-year-ahead growth and longevity forecasts can be sufficiently reliable. Given the actual difficulty in forecasting growth and longevity over a period exceeding 18 months,[7] the swap is rolled over every 10 years. This also allows the parties to take into account sovereign default risk: If country creditworthiness declines, then the swap spread can be raised at the time the contract is rolled over. But as with any rolling strategy, it is important to account for possible rolling or tracking errors and their severity.

The main advantage for the rich country in this fixed-for-floating swap is to transform future cash-outs at a floating rate - its elderly-related expenses which depend on a stochastic population ageing rate - into payments at a fixed rate, thus locking in a "sustainable" population ageing rate. As for the poor country, its main advantage is to hedge against adverse changes in the development aid it receives due to population ageing in the donor country. A long-term contract as this 30-year swap entails, though, the risk of significant fluctuations in exchange rates and interest rates. These risks may be hedged together by adding a (fixed-for-floating) cross-currency interest-rate swap, to be rolled over during the 30-year lifetime of the swap.

---

[6]The objective of commercial microfinance is to increase the set of funding alternatives available to poor countries and microfinance institutions beyond the plain intergovernmental loans they more typically receive. As an example, Bystrom [Bys08] studies microfinance collateralized loan obligations as a tool for economic development. Another type of product are microfinance investment funds.

[7]See Isiklar and Lahiri [IL07].

A couple of benefits of this intergenerational cross-country swap over a simple developmental loan from rich to poor are particularly relevant. One first benefit is the possibility for both parties to spread the cash out- and inflows over time. Secondly, the swap provides the possibility of including a rebate in case the contract is interrupted because of sovereign default or economic recession in either country. But this swap also offers advantages over a longevity derivative between two counterparties based in the rich country. The problem of finding a counterparty willing to take on the long-longevity side of the transaction is often cited by practitioners as the main problem currently hindering the take-off of longevity derivatives. We will show in what follows that a poor country may be willing to assume the longevity risk of a richer country if this risk is forecast to adversely affect the developmental support it receives from the rich country.

The issue of identifying the variable on which to base the swap's cash flows is not trivial, given that no existing measure to date serves our purpose fully and satisfactorily. We need a rate that reveals the strength with which longevity trends impact each country's wealth endowments.

Gross domestic product (GDP) is by far the most widely-used indicator of economic growth, but it does have its shortcomings, too. A crucial shortcoming of GDP measures is that they do not take into account life expectancy or other demographic variables such as fertility, which are important indicators of a country's well-being. What is important for the purpose of this study, however, is to quantify the burden of elderly-related expenses on an economy. These expenses tend to actually raise GDP figures through an increase in government expenditures; but, as discussed above, longevity is forecast to negatively affect the economy of the more advanced economies.

Several studies in growth theory attempt to come up with a growth measure reflecting the welfare gains from both quality and quantity of life.[8] The aim of longevity-adjusted growth rates is to quantify the extent to which national economic growth is affected by population age structure. The literature on the relationship between income growth and life expectancy has taken off in the seventies following studies by Usher [Ush73] and Preston [Pre75]. There have been several country-specific applications since then; e.g. Ponthiére [Pon08] applies the Usher-Williamson-Miller framework to include longevity data into Belgian national income accounting. In a major study in this field, Becker *et al.* [BPS06] develop a statistical model that accounts for the impact of longevity on the evolution of welfare across almost 100 countries from 1960 to 2000. Their model measures the growth of individual income plus the value placed on the growth of an individuals life expectancy. A common result of all these studies is that, by not taking into account increases in longevity, GDP underestimates the extent to which developing countries are gaining relative to developed countries.

One indicator of the economic burden of rising population longevity is the old-age dependency ratio. This gives the number of retirees in percentage of the total population in working age, i.e.

$$\text{old-age dependency ratio} = \frac{\text{population aged 65+}}{\text{population aged 15-64}}.$$

[8] See Becker *et al.* [BPS06].

The evolution of this ratio depends on future trends in fertility, mortality and international migration. A higher forecast dependency ratio is a clear signal of an increasing burden of ageing-related expenses and of the need for governments to put in place the necessary budget reforms. Given the availability of historical and forecast data on the old-age dependency ratio for several countries and their widespread use in policy reform discussions, we choose this variable as the underlying of our swap.[9]

Having identified the underlying and the main features of the swap, we need to guarantee that such a long-term contract does not neglect the risk of sovereign default. A knock-out feature or credit trigger acts as an insurance against sovereign default and causes the payments from rich to poor to be interrupted as soon as the creditworthiness of the poor country falls below a certain level. In a worst-case scenario, the payments from rich to poor cease within a few years and can be taken as a charitable payment.

3. Pricing the swap

Given the innovative nature of our swap's structure, we need to price in an incomplete market. The literature on pricing derivatives when risk-neutral valuation is not sufficient is vast. We here follow a utility-based valuation approach, which relies on the individual rationality requirement of the agent's expected utility from participation in the contract to (weakly) exceed his reservation utility. We take the exponential utility function, which implies constant relative risk aversion and linear risk tolerance. This utility function is widely employed in the literature because of its mathematically tractability. For the mathematical details of exponential utility indifference valuation we refer to Mania and Schweizer [MS05] and Frei and Schweizer [FS08].

In a complete financial market every contingent claim can be perfectly replicated by a portfolio of traded securities and therefore admits a unique arbitrage-free price. In an incomplete market, to every contingent claim is associated an interval of arbitrage-free prices and arbitrage arguments alone are not sufficient to lead to a unique price, i.e. to a replication strategy. As the lower and upper endpoints of this interval coincide with the sub- and super-replication costs of the contingent claim, respectively, any price in the middle will lead to a possible profit & loss at maturity. Hence, the choice of an arbitrage-free price must be made with respect to another criterion.

The utility indifference pricing method has been proved to provide a narrower range of price bounds than the arbitrage-free pricing method.[10] The pricing of the swap is based on the condition of individual rationality for both parties to enter the deal. The participation constraints lead to an interval of prices acceptable to both the seller and the buyer.

We make use of the following notation:

[9] The United Nations Population Division (UNPD) publishes forecasts of the old-age dependency ratio for most countries in the world every year. The most recent population projects are reported in [UNP07]. To project the population until 2050, the UNPD relies on a set of assumptions regarding future trends in fertility, mortality, and international migration. Because future trends cannot be known with certainty, several projection variants are produced. We refer to the UN study [UNP07] for data on historical and forecast old-age dependency ratios around the world according to the medium, high, and low projection variants.

[10] See Mania and Schweizer [MS05].

- W_t denotes the wealth endowment of the rich government at time t - national budget set aside for pensions and long term health care - adjusted for reference population size and inflation;
- $\tilde{W}_t$ denotes the wealth endowment of the poor government at time t - national budget set aside for infrastructure projects - adjusted for reference population size and inflation;
- s_{ask} denotes the fixed ask swap rate;
- s_{bid} denotes the fixed bid swap rate;
- $t = 0, 1, \dots, T$ denotes the time frame, with T the contract maturity;
- $U(W)$ denotes the utility function;
- $D_t(W_t)$ denotes the discount factor;
- $\mathbb{P}^W$ denotes the probability law of the wealth dynamics;
- $\mathbb{P}^a$ denotes the probability law of the old-age dependency ratio dynamics.

Note that we use the tilde notation ˜ for parameters related to the poor country.

The indifference valuation criterium demands that the investor valuing a contingent claim should achieve the same expected utility both in case he does not possess the claim and in case he does possess the claim but his initial capital is reduced by the amount of indifference value of the claim. In this framework, rich and poor country must be made indifferent between bearing longevity and income growth risks without entering a swap agreement under a medium population projection variant and no longer bearing these risks after entering the swap agreement. Thus, the indifference values of the swap to the rich and the poor country are defined through

$$V\left(W_0, s_{bid}, a_0\right) \geq V\left(W_0, 0, 0\right)$$

and

$$\tilde{V}\left(\tilde{W}_0, s_{ask}, a_0\right) \geq \tilde{V}\left(\tilde{W}_0, 0, 0\right),$$

respectively.

The developed economy (or fixed leg or buyer of the swap) pays fixed value and receives realized value at each settlement date, so that its participation constraint is defined by

(3.1)

$$\mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}\left[\sum_{t=0}^{T} D_t\left(W_t\right) * U\left(W_t, s_{bid}, a_t\right)\right] \geq \mathbb{E}_0^{\mathbb{P}_W}\left[\sum_{t=0}^{T} D_t\left(W_t\right) * U\left(W_t, 0, 0\right)\right].$$

This gives the maximum bid price. We simplify the pricing framework by assuming that the government spends its entire budget; hence, we do not include a savings rate.

The developing economy (or floating leg or seller of the swap) pays realized value and receives fixed value at each settlement date, so that its participation constraint is defined by

(3.2)

$$\mathbb{E}_0^{\mathbb{P}_{\tilde{W}} \times \mathbb{P}_a}\left[\sum_{t=0}^{T} D_t\left(\tilde{W}_t\right) * U\left(\tilde{W}_t, s_{ask}, a_t\right)\right] \geq \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\sum_{t=0}^{T} D_t\left(\tilde{W}_t\right) * U\left(\tilde{W}_t, 0, 0\right)\right].$$

This gives the minimum ask price. These two participation constraints allow us to derive a range of prices within which the swap price (i.e. the fixed swap rate) needs to lie.

We describe social welfare by the negative exponential utility function:

$$U\left(x_t\right) = -\,\mathrm{e}^{-\lambda x_t}, \quad \lambda \in (0, \infty). \tag{3.3}$$

Each country discounts its future utility according to its specific social discount rate. The discounted marginal utility of government income is specified by

$$u = U'\left(W_t\right)\,\mathrm{e}^{-\rho t} = \lambda\,\mathrm{e}^{-\lambda W_t}\,\mathrm{e}^{-\rho t}$$

and the social discount rate by

$$d_t = -\frac{\dot{u}}{u} = \rho + \lambda \dot{W}_t \approx \rho + \lambda * \left(W_t - W_{t-1}\right), \tag{3.4}$$

where ρ denotes the social rate of time preference, representing the value society attaches to present consumption relative to future consumption. Hence, the social discount factor is given by

$$D_t\left(W_t\right) = \mathrm{e}^{-d_t t} = \mathrm{e}^{-\rho t - \lambda * (W_t - W_{t-1}) t}. \tag{3.5}$$

The environmental finance literature has highlighted several reasons why it is inappropriate to simply use market risk-free long-term interest rates, such as the rates on government bonds with equivalent maturity.[11] The primary reason is intergenerational concerns, which a government is supposed to take into account when initiating projects that are deemed to affect future generations. The importance given to future generations depends on current wealth and expected future wealth levels. If age structure changes are expected to slow down future economic growth rates, the social discount rate should decline accordingly. This translates into greater sacrifices on behalf of the current generation to take into account future generations' well-being. Pearce *et al.* [PGHK03] argue that in the extreme case of projected long-term recession, the social discount rate should be negative.[12]

Proposition 3.1. *Consider the two participation constraints (3.1) and (3.2), the utility function (3.3), and the discount factor (3.5). Then the fixed swap rate, s, must lie within a range with upper bound*

$$\begin{aligned} \overline{s}_{bid} \;&=\; \frac{1}{\lambda}\left(\ln \sum_{t=0}^{T} \mathrm{e}^{-\rho t}\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda W_t(1+t)+\lambda W_{t-1}t}\right]\right. \\ &\quad \left. -\ln \sum_{t=0}^{T} \mathrm{e}^{-\rho t}\mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda W_t(1+t)+\lambda W_{t-1}t-\lambda a_t}\right]\right) \end{aligned} \tag{3.6}$$

and lower bound

$$\begin{aligned} \underline{s}_{ask} \;&=\; \frac{1}{\tilde{\lambda}}\left(\ln \sum_{t=0}^{T} \mathrm{e}^{-\tilde{\rho} t}\mathbb{E}_0^{\mathbb{P}_{\tilde{W}}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\lambda}\tilde{W}_t(1+t)+\tilde{\lambda}\tilde{W}_{t-1}t+\tilde{\lambda}a_t}\right]\right. \\ &\quad \left. -\ln \sum_{t=0}^{T} \mathrm{e}^{-\tilde{\rho} t}\mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\lambda}\tilde{W}_t(1+t)+\tilde{\lambda}\tilde{W}_{t-1}t}\right]\right). \end{aligned} \tag{3.7}$$

---

[11] See Pearce *et al.* [PGHK03].

[12] The question whether society should place a lower value on a future gain or loss than on the same gain or loss occurring now is highly controversial. A large body of literature analyzes social discounting for environmental policies and how governments should value climate change damages; see, among others, Weitzman [Wei01], Weitzman [Wei07], Pearce *et al.* [PGHK03], and Hepburn [Hep06]. Application examples include the *Stern review on the economics of climate change* and the cost-benefit analyses employed by the Copenhagen Consensus to examine solutions to ten of the world's biggest challenges.

Proof. See appendix section A.1. □

Equations (3.6) and (3.7) show that the price bounds are a function of λ, $\mathbb{E}_0^{\mathbb{P}_W}[W_t]$, and $\mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}[W_t, a_t]$ for the bid price; $\tilde{\lambda}$, $\mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\tilde{W}_t\right]$, and $\mathbb{E}_0^{\mathbb{P}_{\tilde{W}} \times \mathbb{P}_a}\left[\tilde{W}_t, a_t\right]$ for the ask price. Furthermore, these price bounds need to satisfy a couple of conditions. First of all, both the minimum price the seller is willing to receive for the swap and the maximum price the buyer is willing to pay for the swap must be positive. Secondly, the maximum bid price should not be lower than the minimum ask price; otherwise, no agreement between the two parties is possible.

The next step in pricing the swap is to define the stochastic paths of the three sources of uncertainty, a_t, W_t, and $\tilde{W}_t$. Forecasts for years 2010 up to 2030 show a clear upward trend in population ageing in the industrial countries, though this trend may experience ‘up’ or ‘down’ episodes, for example, due to extreme heat waves. We do not get into the details of longevity forecasting, but nevertheless highlight a few contentious points, such as whether and how medical advances should be taken into account, whether or not there exists a biological limit to life, and whether lifespans could eventually be extended through genetic changes or non-genetic interactions. Summing up, we may represent this series by the following upward-trending process:

$$a_t = a_0 + \delta t + \epsilon_t^a, \tag{3.8}$$

where the error term, ϵ_t^a, follows an autocorrelation process with a lag of order k:

$$\epsilon_t^a = \sum_{i=1}^{k} \varsigma_t \epsilon_{t-i}^a + \sigma^a \xi_t^a, \quad \xi^a \sim \mathcal{N}(0,1).$$

Fitting this process to historical and forecast data for Switzerland for years 1991 to 2050, we have derived parameter values $a_0 = 23.4$ and $\delta = 0.45$.[13] In a future companion paper, we will estimate the parameters of the autocorrelation process and the lag order for a number of selected countries.

Evolution of the wealth endowment over time in the developed country can be thought to follow a mean-reverting process, converging to some long-term value. The wealth level is adjusted for inflation and for population growth, and so is expressed in real per-capita terms. This value of mean reversion can be thought of as a proportion of national income the government desires to devote to elderly-related expenses, e.g. long-term health care. Hence, this process needs to mirror the evolution of per-capita national income in real terms.

We may take the following AR(1) process:

$$W_t = \alpha + \beta W_{t-1} + \epsilon_t, \quad 0 \leq \beta < 1, \tag{3.9}$$

where $1 - \beta$ measures the speed of mean reversion in the rate of wealth changes. The error term is given by

$$\epsilon_t = \sigma \xi_t + \sigma \sum_{i=0}^{m} \eta_i \xi_{t-i}^a, \quad \xi, \xi^a \sim \mathcal{N}(0,1). \tag{3.10}$$

with $0 \leq m \leq +\infty$ and where ξ_t captures the randomness due to all factors other than the uncertainty related to the population age structure; $\xi_t^a, \ldots, \xi_{t-m}^a$ are the

[13] We have used publicly-available data from the Bundesamt für Statistik and the State Secretariat for Economic Affairs.

current and lagged innovations of the old-age dependency ratio; η_t are the sensitivity parameters between the wealth process and population ageing. In the extreme case of W and a being independent, i.e. when population ageing has no effect on the wealth endowment, the sensitivity parameters η_t are all equal to zero, so that the error term is simply an i.i.d. standard normal random variable, i.e. $\epsilon_t = \sigma\xi_t$. The solution to (3.9) is

$$W_t = \alpha \sum_{i=0}^{t-1} \beta^i + \beta^t W_0 + \sum_{i=0}^{t-1} \beta^i \epsilon_{t-i}. \tag{3.11}$$

For the developing economy we take the following AR(1) process:

$$\tilde{W}_t = \tilde{\alpha} + \tilde{\beta}\tilde{W}_{t-1} + \tilde{\gamma}t + \tilde{\epsilon}_t, \quad 0 \leq \tilde{\beta} < 1,\ \gamma > 0, \tag{3.12}$$

where, once again, $1 - \tilde{\beta}$ measures the speed of mean reversion and the error term is given by

$$\tilde{\epsilon}_t = \tilde{\sigma}\tilde{\xi}_t + \tilde{\sigma} \sum_{i=0}^{m} \tilde{\eta}_i \xi^a_{t-i}, \quad \tilde{\xi}, \xi^a \sim \mathcal{N}(0,1). \tag{3.13}$$

The term $\tilde{\gamma}t$ allows for a small upward trend which captures economic growth of the developing economy. The solution to (3.12) is

$$\tilde{W}_t = \tilde{\alpha} \sum_{i=0}^{t-1} \tilde{\beta}^i + \tilde{\beta}^t \tilde{W}_0 + \tilde{\gamma} \sum_{i=0}^{t-1} t\tilde{\beta}^i + \sum_{i=0}^{t-1} \tilde{\beta}^i \tilde{\epsilon}_{t-i}. \tag{3.14}$$

We may solve the geometric sum in (3.11) and rewrite the equation in a more compact and tractable form as

$$W_t = \varphi_0 + \varphi_1 \beta^t + \sum_{i=0}^{t-1} \beta^i \epsilon_{t-i}, \tag{3.15}$$

with

$$\varphi_0 = \frac{\alpha}{1-\beta}, \quad \varphi_1 = -\frac{\alpha}{1-\beta} + W_0.$$

Similarly for (3.14) we obtain

$$\tilde{W}_t = \tilde{\varphi}_0 + \tilde{\varphi}_1 \tilde{\beta}^t + \tilde{\varphi}_2 t + \sum_{i=0}^{t-1} \tilde{\beta}^i \tilde{\epsilon}_{t-i}, \tag{3.16}$$

with

$$\tilde{\varphi}_0 = \frac{\tilde{\alpha}}{1-\tilde{\beta}}, \quad \tilde{\varphi}_1 = -\frac{\tilde{\alpha}+\tilde{\gamma}}{1-\tilde{\beta}} + \tilde{W}_0, \quad \tilde{\varphi}_2 = \frac{\tilde{\gamma}}{1-\tilde{\beta}}.$$

Inserting (3.14) and (3.11) into the bounds (3.6) and (3.7) gives us explicit expressions for the minimum ask price and maximum bid price, as outlined in the following proposition.

Proposition 3.2. *Under wealth dynamics (3.9) and (3.12) and population ageing (3.8), the fixed swap rate, s, must lie within a range with upper bound*

$$\bar{s}_{bid} = \frac{1}{\lambda}\left(\ln \sum_{t=0}^{T} \Psi_t \mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\Lambda_t}\right] - \ln \sum_{t=0}^{T} \Psi^a_t \mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}\left[\mathrm{e}^{-\Lambda^a_t}\right]\right)$$

and lower bound

$$\underline{s}_{ask} = \frac{1}{\tilde{\lambda}} \left(\ln \sum_{t=0}^{T} \tilde{\Psi}^a_t \mathbb{E}_0^{\mathbb{P}_{\tilde{W}} \times \mathbb{P}_a} \left[\mathrm{e}^{-\tilde{\Lambda}^a_t} \right] - \ln \sum_{t=0}^{T} \tilde{\Psi}_t \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}} \left[\mathrm{e}^{-\tilde{\Lambda}_t} \right] \right),$$

with

$$\Psi_t := \mathrm{e}^{-\rho t} \mathrm{e}^{-\lambda \left[\varphi_0 + \varphi_1 \beta^{t-1} [(1+t)\beta - t]\right]}, \quad \Psi^a_t = \Psi_t \, \mathrm{e}^{-\lambda \delta t},$$

$$\Lambda_t := \lambda \left[(1+t)\, \epsilon_t + t \sum_{i=0}^{t-1} \beta^i \epsilon_{t-i} \right], \quad \Lambda^a_t = \Lambda_t - \lambda \epsilon^a_t,$$

$$\tilde{\Psi}_t := \mathrm{e}^{-\tilde{\rho} t} \mathrm{e}^{-\tilde{\lambda} \left[\tilde{\varphi}_0 + \tilde{\varphi}_1 \tilde{\beta}^{t-1} \left[(1+t)\tilde{\beta} - t\right] + 2\tilde{\varphi}_2 t\right]}, \quad \tilde{\Psi}^a_t = \tilde{\Psi}_t \, \mathrm{e}^{\tilde{\lambda} \delta t},$$

and

$$\tilde{\Lambda}_t := \tilde{\lambda} \left[(1+t)\, \tilde{\epsilon}_t + t \sum_{i=0}^{t-1} \tilde{\beta}^i \tilde{\epsilon}_{t-i} \right], \quad \tilde{\Lambda}^a_t = \tilde{\Lambda}_t + \tilde{\lambda} \tilde{\epsilon}^a_t.$$

The formulas for the price bounds will differ when we assume the two stochastic processes W_t and a_t to be either dependent or independent of each other. The latter case of processes' dependence is obviously more realistic and is precisely what motivates our innovation idea, but it is equally important to check how the price bounds look like in the former case, since only by doing so can we deduce interesting comparative statics.

4. Exponential-utility-based price bounds

Consider first the case where the stochastic processes W_t and a_t are independent.

Proposition 4.1. *Take W_t and a_t independent. Under wealth dynamics (3.9) and (3.12) and population ageing (3.8), the fixed swap rate, s, must lie within a range with upper bound*

$$\overline{s}_{bid} = \frac{1}{\lambda} \left(\ln \sum_{t=0}^{T} \Psi_t \Omega_t - \ln \sum_{t=0}^{T} \Psi^a_t \Omega^a_t \right)$$

and lower bound

$$\underline{s}_{ask} = \frac{1}{\tilde{\lambda}} \left(\ln \sum_{t=0}^{T} \tilde{\Psi}^a_t \tilde{\Omega}^a_t - \ln \sum_{t=0}^{T} \tilde{\Psi}_t \tilde{\Omega}_t \right),$$

with

$$\Psi_t := \mathrm{e}^{-\rho t} \mathrm{e}^{-\lambda \left[\varphi_0 + \varphi_1 \beta^{t-1} [(1+t)\beta - t]\right]}, \quad \Psi^a_t = \Psi_t \, \mathrm{e}^{-\lambda \delta t},$$

$$\Omega_t := \mathrm{e}^{-\frac{\lambda^2}{2} \sigma^2 \left[(1+t)^2 + t^2 \left(\sum_{i=1}^{t-1} \beta^i \right)^2 \right]}, \quad \Omega^a_t := \mathrm{e}^{-\frac{\lambda^2}{2} \left[\sigma^2 \left[(1+t)^2 + t^2 \left(\sum_{i=1}^{t-1} \beta^i \right)^2 \right] + k \varsigma_t^2 + (\sigma^a)^2 \right]},$$

$$\tilde{\Psi}_t := \mathrm{e}^{-\tilde{\rho} t} \mathrm{e}^{-\tilde{\lambda} \left[\tilde{\varphi}_0 + \tilde{\varphi}_1 \tilde{\beta}^{t-1} \left[(1+t)\tilde{\beta} - t\right] + 2\tilde{\varphi}_2 t\right]}, \quad \tilde{\Psi}^a_t = \tilde{\Psi}_t \, \mathrm{e}^{\tilde{\lambda} \delta t},$$

and

$$\tilde{\Omega}_t := \mathrm{e}^{-\frac{\tilde{\lambda}^2}{2} \tilde{\sigma}^2 \left[(1+t)^2 + t^2 \left(\sum_{i=1}^{t-1} \tilde{\beta}^i \right)^2 \right]}, \quad \tilde{\Omega}^a_t := \mathrm{e}^{-\frac{\tilde{\lambda}^2}{2} \left[\tilde{\sigma}^2 \left[(1+t)^2 + t^2 \left(\sum_{i=1}^{t-1} \tilde{\beta}^i \right)^2 \right] + k \varsigma_t^2 + (\sigma^a)^2 \right]}.$$

Proof. See appendix section A.2. $\square$

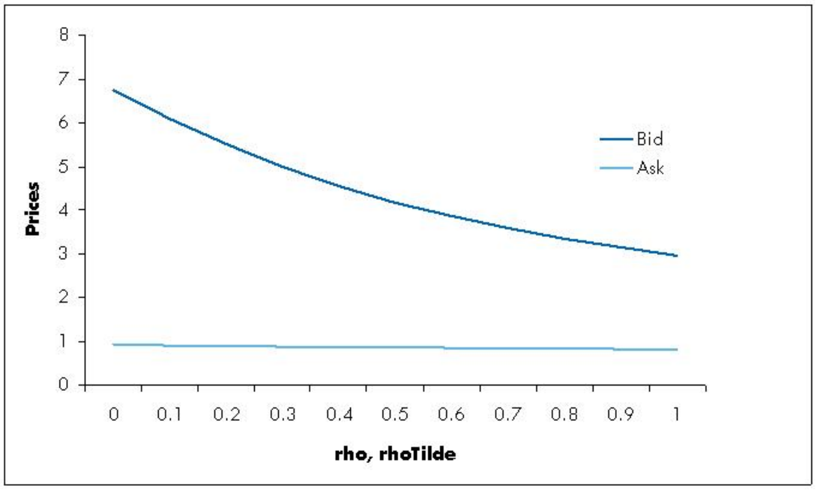

FIGURE 4.1. Changes in the bid-ask spread with respect to changes in ρ and $\tilde{\rho}$. Prices in CHF billions.

So if we assume the wealth endowments in both the rich and poor countries to be totally independent of population ageing in the rich country, the maximum bid price and the minimum ask price are a function of the risk aversion parameters, social discount rates, process parameters for W and a. To satisfy the two conditions of positive price bounds and a non-empty price interval, the following inequalities need to hold:

$$\lambda \delta t \neq 0; \qquad \frac{\lambda^2 k \varsigma_t^2 + (\sigma^a)^2}{2} \neq 0;$$

$$\ln \sum_{t=0}^{T} \Psi_t \Omega_t > \ln \sum_{t=0}^{T} \Psi_t^a \Omega_t^a; \tag{4.1}$$

$$\ln \sum_{t=0}^{T} \tilde{\Psi}_t^a \tilde{\Omega}_t^a > \ln \sum_{t=0}^{T} \tilde{\Psi}_t \tilde{\Omega}_t. \tag{4.2}$$

Whereas condition (4.1) always holds true, condition (4.2) only holds true for T greater than some critical value T^*. Indeed, given the long-term character of the risks faced by the two counterparties, a contract with too short a maturity would not provide a useful hedge against these risks.

Sensitivity analysis allows us to observe whether and how the price interval widens or narrows for different parameter values. To check for the intuition behind the sensitivity analysis and to graphically illustrate the price changes, we apply the model to population and economic data on Switzerland.[14]

First of all, as contract life T increases, both $\overline{s}_{bid}$ and $\underline{s}_{ask}$ increase. So it will not be worthwhile for the rich and poor country to enter a contract with too short a

[14]Without loss of generality and for illustrative purposes, we assume rich and poor countries to have the same risk and time preferences. In future work, it will be interesting to check how our results vary as we take differing parameters for the two counterparties.

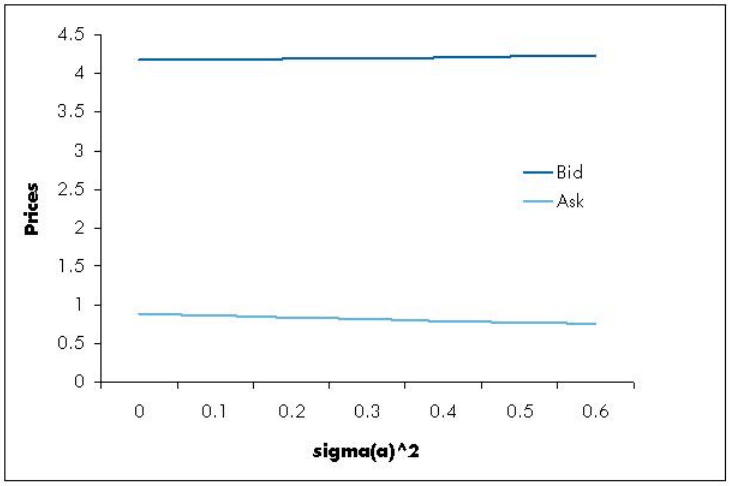

FIGURE 4.2. Changes in the bid-ask spread with respect to changes in $(\sigma^a)^2$. Prices in CHF billions.

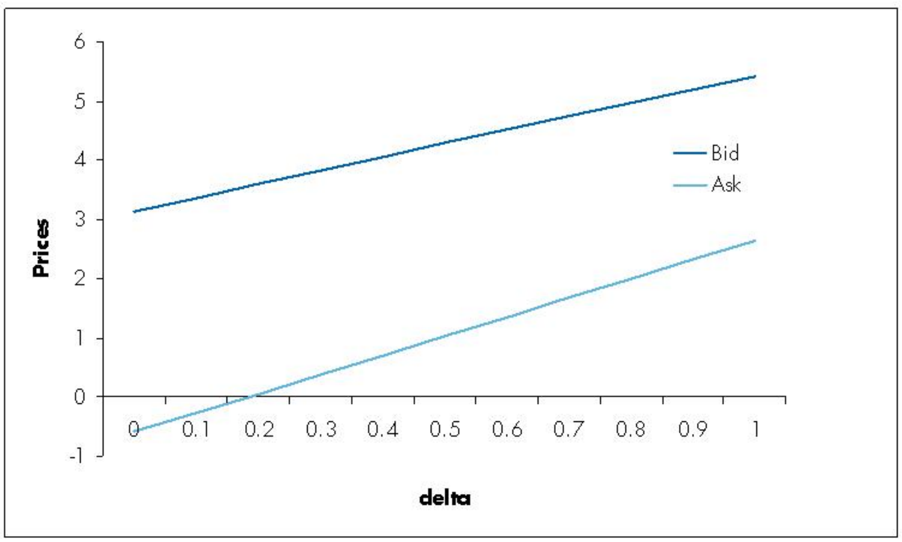

FIGURE 4.3. Changes in the bid-ask spread with respect to changes in δ. Prices in CHF billions.

maturity. This is a rather intuitive result, since the rich country does not forecast to experience longevity-related budget problems until some time in the further future.

For higher social rates of time preference ρ and $\tilde{\rho}$, both countries quote lower prices; however, the decrease in the quoted bid price is much stronger and the price interval narrows considerably with an increase in both countries' discount rates. This is illustrated in figure 4.1. Note, however, that the stronger decrease in the bid price is only true in absolute terms; in relative terms, the decrease in both prices is roughly equally strong.

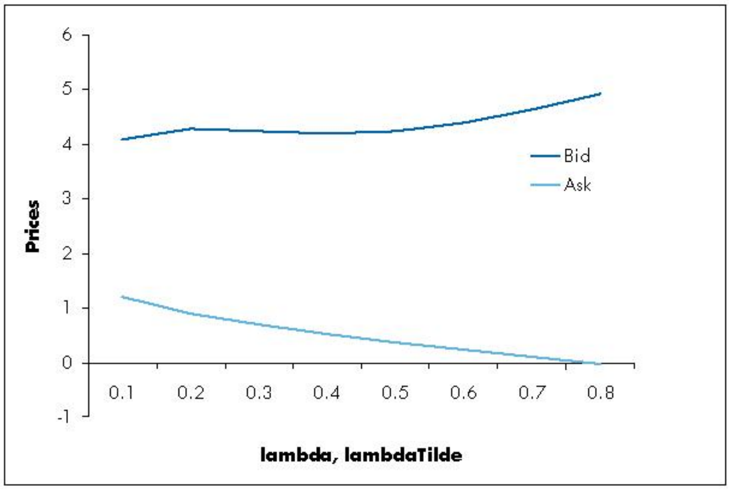

FIGURE 4.4. Changes in the bid-ask spread with respect to changes in λ and $\tilde{\lambda}$. Prices in CHF billions.

For an increasing volatility in the innovations of the population ageing process, σ^a, both $\overline{s}_{bid}$ and $\underline{s}_{ask}$ increase, but rather mildly. Therefore, as figure 4.2 illustrates, the price interval does not change significantly.

If the old-age dependency ratio is expected to increase at a faster pace, i.e. for a higher value of δ, both the seller and the buyer of the swap will quote higher prices. As figure 4.3 illustrates, there is a critical value of δ which guarantees that the seller will quote a positive price; whereas, for too low a δ value, the seller will not be interested in entering such a contract. This result confirms precisely our intuition behind such a financial innovation, since the swap only makes economic sense if there is a severe population ageing risk to be hedged.

The more interesting result from the sensitivity analysis is with regards to the rich country's risk aversion parameter. Figure 4.4 shows indeed a kink in the shape of the bid price curve. This shape points out to the country being more interested in hedging its risk of severe population ageing either when it is relatively little risk averse or very risk averse. We believe that this interesting shape of the bid price curve is worthy of further investigation.

We now take the two processes W_t and a_t to be dependent of each other and check how the price interval varies as the interdependence between the two processes comes into play.

Proposition 4.2. *Take W_t and a_t dependent as defined in (3.10). Under wealth dynamics (3.9) and (3.12) and population ageing (3.8), the fixed swap rate, s, must lie within a range with upper bound*

$$\overline{s}_{bid} = \frac{1}{\lambda}\left(\ln\sum_{t=0}^{T}\Psi_t\Omega_t - \ln\sum_{t=0}^{T}\Psi_t^a\Omega_t^a\right)$$

and lower bound

$$\underline{s}_{ask} = \frac{1}{\tilde{\lambda}}\left(\ln\sum_{t=0}^{T}\tilde{\Psi}_t^a\tilde{\Omega}_t^a - \ln\sum_{t=0}^{T}\tilde{\Psi}_t\tilde{\Omega}_t\right),$$

with

$$\Psi_t := \mathrm{e}^{-\rho t}\mathrm{e}^{-\lambda\left[\varphi_0+\varphi_1\beta^{t-1}[(1+t)\beta-t]\right]}, \quad \Psi_t^a = \Psi_t\,\mathrm{e}^{-\lambda\delta t},$$

$$\Omega_t := \mathrm{e}^{\frac{\left[-\lambda(1+t)\sigma-\lambda(1+t)\sigma\sum_{j=0}^m \eta_j-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^m\eta_j\right]^2}{2}},$$

$$\Omega_t^a := \mathrm{e}^{\frac{\left[-\lambda(1+t)\sigma-\lambda(1+t)\sigma\sum_{j=0}^m \eta_j-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^m\eta_j-\lambda\sum_{i=1}^k\varsigma_t-\lambda\sigma^a\right]^2}{2}},$$

$$\tilde{\Psi}_t := \mathrm{e}^{-\tilde{\rho}t}\mathrm{e}^{-\tilde{\lambda}\left[\tilde{\varphi}_0+\tilde{\varphi}_1\tilde{\beta}^{t-1}\left[(1+t)\tilde{\beta}-t\right]+2\tilde{\varphi}_2 t\right]}, \quad \tilde{\Psi}_t^a = \tilde{\Psi}_t\,\mathrm{e}^{\tilde{\lambda}\delta t},$$

$$\tilde{\Omega}_t := \mathrm{e}^{\frac{1}{2}\left[-\tilde{\lambda}(1+t)\tilde{\sigma}-\tilde{\lambda}(1+t)\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_j-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_j\right]^2},$$

and

$$\tilde{\Omega}_t^a := \mathrm{e}^{\frac{1}{2}\left[-\tilde{\lambda}(1+t)\tilde{\sigma}-\tilde{\lambda}(1+t)\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_j-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}-\tilde{\lambda}t\sum_{i=1}^{t-1}\beta^i\tilde{\sigma}\sum_{j=0}^m\eta_j+\tilde{\lambda}\sum_{i=1}^k\varsigma_t+\tilde{\lambda}\sigma^a\right]^2}.$$

Proof. See appendix section A.3. □

The inherent market price of risk for the old age dependency ratio results from the inability to hedge the risk of an ageing population. I.e. the interrelationship between population ageing in the developed country and the wealth endowments in both countries plays an important role in determining the risk premium embedded in the value of this contract.

If the rich country's wealth endowment is very sensitive to the ageing of its population, then it is more interested in entering the swap agreement and is willing to pay more to hedge against adverse demographic developments. This raises the maximum bid quote. Similarly, if the poor country's wealth endowment is very sensitive to the ageing of the rich country's population, then it is more interested in entering the swap agreement and is willing to receive less to hedge against adverse demographic developments. This is especially the case if the poor country is highly dependent on financial aid from the rich country and is at risk of receiving less aid if the rich country needs to devote extra funds to its own elderly population. It may be interesting to check the extent to which developmental aid flows are or may be in the future affected by population ageing in the donor countries.

The amount of the risk premium can also be shown to depend on the financial position of both countries, too. Not only does the sensitivity of the wealth endowment matter, but also the amount of government income invested in the endowment. Higher values of W_0 and $\tilde{W}_0$ raise φ_1 and $\tilde{\varphi}_1$, thus shifting the bid-ask interval towards lower prices. So countries entering the swap agreement with higher wealth are likely to demand a lower risk premium.

5. Challenges

The swap we have presented throughout the previous sections does not yet exist. We may think of three significant challenges arising when implementing this intergenerational cross-country swap. First of all, what percentage of its GDP would the rich country be willing to pay for longevity protection? Given our model assumptions, together with historical GDP and forecast longevity data for Switzerland, we estimate the maximum bid price to lie between 3 to 5 billion Swiss francs, which is roughly not more than 1% of current real Swiss GDP.[15] We need to compare this figure with the amount of funds the Swiss government forecasts to spend on pensions and long-term health care over the next three decades. According to the base scenario outlined in the Swiss Health Observatory study [SBJR+08], long-term

[15] Data as of end of 2008, according to the Swiss Secretariat for Economic Affairs.

health care costs will rise by 141.9% from 2005 to 2030 to 17.8 billion Swiss francs (keeping 2005 prices constant); this is equivalent to 2.8% of Swiss GDP. Though this may hint to the feasibility of the swap, the decision ultimately remains a political decision.

Another challenge concerns the impact that a transaction of such magnitude could have on currency markets. The announcement of very large sums of money denominated in a certain currency flowing from one country to the other may significantly destabilize exchange rates. Given the large sums already exchanged by central banks through FX swaps when conducting their monetary policies, it is to be checked how much "capacity" the FX markets are still able to bare at the time of contract conclusion.

Finally, we have previously mentioned the risks entailed by rolling over a derivative contract. These are related to the agent's regret in case the bid-ask spread in one period is far above or below the spread in the previous period; if applying backward induction, there is the risk that one of the counterparties will decide not to participate. Take, for example, the buyer of the swap: if the bid-ask spread in the second period is such that the minimum ask price lies above the maximum bid price quoted in the first period, then - through backward induction - the buyer would have been better off not entering the contract in the first place. This rollover risk cannot be hedged and, as such, is *per se* an interesting matter worthy of future analysis.

6. Conclusions and future research

The question we set off to answer is whether there is a rationale for a financial innovation whereby a developed economy would swap its longevity risk against the growth risk of a developing economy. Our answer is yes. Having identified the old-age dependency ratio as the appropriate underlying variable, we proceeded to price the innovative swap structure. We applied an exponential-utility-based pricing method and determined an interval of swap prices, any one of which makes an agreement between the two countries plausible. Yet this begs the question of which final price between maximum bid price and minimum ask price will actually be chosen. As this swap is an over-the-counter contract, liquidity issues are here irrelevant. So what is crucial to consider at this stage is the political strength of the two countries; agreements of international finance are notoriously guided by political issues which cannot be captured by financial pricing models. Intuitively though, the final swap price will converge to the minimum ask price as the rich country's political strength relative to the poor country increases. Similarly, the final swap price will converge to the maximum bid price as the rich country's political strength relative to the poor country decreases. We could include some parameter to illustrate this basic intuition, but this would not induce any significant changes to the results derived in the paper.

In a following paper we will estimate the dynamics W and a for a selected group of countries and check which countries could benefit from such an intergenerational agreement. We aim to quantify the swap rates by considering the participation constraints under alternative scenarios of demographic changes. Other issues we aim to investigate in future work include the channels through which banks' product strategies are influenced by asymmetric demographic trends, as well as the extent to which developmental aid and microfinance innovations are related to trends in population ageing in donor countries.

References

[AB07] J. Alho and V. Borgy. Global ageing and macroeconomic consequences of demographic uncertainty in a multi-regional model. Working paper, CEPII, 2007.

[Abe03] A. Abel. The effects of a baby boom on stock prices and capital accumulation in the presence of social security. *Econometrica*, 71:551–595, 2003.

[AS01] S. Athanasoulis and R. Shiller. World income components: Measuring and exploiting risk sharing opportunities. *American Economic Review*, 91:103154, 2001.

[BB01] D. Blake and W. Burrows. Survivor bonds: Helping to hedge mortality risk. *Journal of Risk and Insurance*, 68:339–348, 2001.

[BCDM06] D. Blake, A. Cairns, K. Dowd, and R. MacMinn. Longevity bonds: Financial engineering, valuation, and hedging. *Journal of Risk and Insurance*, 73:647–672, 2006.

[BCM06] N. Batini, T. Callen, and W. McKibbin. The global impact of demographic change. Working paper, International Monetary Fund, 2006.

[BPS06] G. Becker, T. Philipson, and R. Soares. The quantity and quality of life and the evolution of world inequality. Working paper, NBER, 2006.

[Bro02] R. Brooks. Asset-market effects of the baby boom and social security reform. *American Economic Review, Papers and Proceedings*, 192:402–406, 2002.

[Bry06] R. Bryant. Asymmetric demography and macroeconomic interactions across national borders. In *Demography and Financial Markets* [].

[Bys08] H. Byström. The microfinance collateralized debt obligation: A modern Robin Hood? *World Development*, 36:2109–2126, 2008.

[DBCD06] K. Dowd, D. Blake, A. Cairns, and P. Dawson. Survivor swaps. *Journal of Risk and Insurance*, 73:1–17, 2006.

[EW03] S. Eijffinger and W. Wagner. The feasible gains from international risk sharing. Working paper, CESifo, 2003.

[FS08] C. Frei and M. Schweizer. Exponential utility indifference valuation in a general semi-martingale model. Working paper, NCCR FinRisk, 2008.

[GBH+06] T. Groome, N. Blancher, F. Haas, J. Kiff, W. Lee, P. Mills, S. Nakagawa, P. Ramlogan, O. Khadarina, and Y. Kim. The limits of market-based risk transfer and implications for managing systemic risks. Working paper, International Monetary Fund, 2006.

[GBR06] T. Groome, N. Blancher, and P. Ramlogan. Population ageing and financial markets. *Finance and Development*, 43:69–81, 2006.

[GMU07] B. Gadanecz, R. Moessner, and C. Upper. Economic derivatives. *BIS Quarterly Review*, pages 69–81, 2007.

[Hep06] S. Hepburn. Discounting climate change damages. Overview for the Stern Review, 2006.

[IL07] G. Isiklar and K. Lahiri. How far ahead can we forecast? Evidence from cross-country surveys. *International Journal of Forecasting*, 23:167–187, 2007.

[Kra02] M. Kraemer. Western europe past its prime - Sovereign rating perspectives in the context of ageing populations. Technical report, Standard and Poor's, 2002.

[MPSY06] O. Mitchell, J. Piggott, M. Sherris, and S. Yow. Financial innovation for an ageing world. In *Demography and Financial Markets* [].

[MS05] M. Mania and M. Schweizer. Dynamic exponential utility indifference valuation. *Annals of Applied Probability*, 15:2113–2143, 2005.

[PGHK03] D. Pearce, B. Groom, C. Hepburn, and P. Koundouri. Valuing the future: Recent advances in social discounting. *World Economics*, 4:121–141, 2003.

[Pon08] G. Ponthière. A study of the sensitivity of longevity-adjusted income measures. *Oxford Development Studies*, 36:339–361, 2008.

[Pre75] S. Preston. The changing relation between mortality and level of economic development. *Population Studies*, 29:231–248, 1975.

[SBJR+08] L. Seematter-Bagnoud, J. Junod, H. Jaccard Ruedin, M. Roth, C. Foletti, and B. Santos-Eggimann. Angebot und Inanspruchnahme ambulanter medizinischer Leistungen in der Schweiz - Prognosen bis zum Jahr 2030. Technical report, Swiss Health Observatory, 2008.

[Shi04a] R. Shiller. *A New Financial Order*. Princeton University Press, 2004.
[Shi04b] R. Shiller. Radical financial innovation. In J. Hakala and U. Wystup, editors, *Entrepreneurship, Innovation and the Growth Mechanism of the Free Market Economies*. Princeton University Press, 2004.
[UNP07] UNPD. World population prospects - The 2006 revision. Technical report, United Nations Population Division, 2007.
[Ush73] D. Usher. An imputation of the measure of economic growth for changes in life expectancy. *Studies in Income and Wealth*, 38:193–225, 1973.
[Wei01] M. Weitzman. Gamma discounting. *American Economic Review*, 91:260–271, 2001.
[Wei07] M. Weitzman. The Stern review of the economics of climate change. *Journal of Economic Literature*, 45:703–724, 2007.

Appendix A. Proofs

This section contains detailed proofs to all propositions in the paper.

A.1. Proof of proposition 3.1.

Proof.

$$\begin{aligned}
\mathbb{E}_0^{\mathbb{P}_{\tilde W}\times\mathbb{P}_a}\left[\sum_{t=0}^{T} D_t\left(\tilde W_t\right) * U\left(\tilde W_t, s_{ask}, a_t\right)\right] &\geq \mathbb{E}_0^{\mathbb{P}_{\tilde W}}\left[\sum_{t=0}^{T} D_t\left(\tilde W_t\right) * U\left(\tilde W_t, 0, 0\right)\right]\\
\mathbb{E}_0^{\mathbb{P}_{\tilde W}\times\mathbb{P}_a}\left[-\sum_{t=0}^{T} \mathrm{e}^{-\tilde\rho t-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t}\,\mathrm{e}^{-\tilde\lambda\left(\tilde W_t+s_{ask}-a_t\right)}\right] &\geq \mathbb{E}_0^{\mathbb{P}_{\tilde W}}\left[-\sum_{t=0}^{T} \mathrm{e}^{-\tilde\rho t-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t}\,\mathrm{e}^{-\tilde\lambda \tilde W_t}\right]\\
-\mathrm{e}^{-\tilde\lambda s_{ask}}\sum_{t=0}^{T}\mathbb{E}_0^{\mathbb{P}_{\tilde W}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde\rho t-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t-\tilde\lambda\left(\tilde W_t-a_t\right)}\right] &\geq -\sum_{t=0}^{T}\mathbb{E}_0^{\mathbb{P}_{\tilde W}}\left[\mathrm{e}^{-\tilde\rho t-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t-\tilde\lambda \tilde W_t}\right]
\end{aligned}$$

so that

$$\begin{aligned}
\mathrm{e}^{-\tilde\lambda s_{ask}} &\leq \frac{\sum_{t=0}^{T}\mathbb{E}_0^{\mathbb{P}_{\tilde W}}\left[\mathrm{e}^{-\tilde\rho t-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t-\tilde\lambda \tilde W_t}\right]}{\sum_{t=0}^{T}\mathbb{E}_0^{\mathbb{P}_{\tilde W}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde\rho t-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t-\tilde\lambda\left(\tilde W_t-a_t\right)}\right]}\\
\mathrm{e}^{-\tilde\lambda s_{ask}} &\leq \frac{\sum_{t=0}^{T}\mathrm{e}^{-\tilde\rho t}\mathbb{E}_0^{\mathbb{P}_{\tilde W}}\left[\mathrm{e}^{-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t-\tilde\lambda \tilde W_t}\right]}{\sum_{t=0}^{T}\mathrm{e}^{-\tilde\rho t}\mathbb{E}_0^{\mathbb{P}_{\tilde W}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t-\tilde\lambda\left(\tilde W_t-a_t\right)}\right]}\\
-\tilde\lambda s_{ask} &\leq \ln\frac{\sum_{t=0}^{T}\mathrm{e}^{-\tilde\rho t}\mathbb{E}_0^{\mathbb{P}_{\tilde W}}\left[\mathrm{e}^{-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t-\tilde\lambda \tilde W_t}\right]}{\sum_{t=0}^{T}\mathrm{e}^{-\tilde\rho t}\mathbb{E}_0^{\mathbb{P}_{\tilde W}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde\lambda *\left(\tilde W_t-\tilde W_{t-1}\right)t-\tilde\lambda\left(\tilde W_t-a_t\right)}\right]}\\
s_{ask} &\geq \frac{1}{\tilde\lambda}\left(\ln\sum_{t=0}^{T}\mathrm{e}^{-\tilde\rho t}\mathbb{E}_0^{\mathbb{P}_{\tilde W}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde\lambda\tilde W_t(1+t)+\tilde\lambda\tilde W_{t-1}t+\tilde\lambda a_t}\right]-\ln\sum_{t=0}^{T}\mathrm{e}^{-\tilde\rho t}\mathbb{E}_0^{\mathbb{P}_{\tilde W}}\left[\mathrm{e}^{-\tilde\lambda\tilde W_t(1+t)+\tilde\lambda\tilde W_{t-1}t}\right]\right),
\end{aligned}$$

which gives the minimum price the seller is willing to receive for the swap.

From the buyer's side:

$$
\begin{aligned}
\mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}\left[\sum_{t=0}^{T} D_t(W_t) * U\left(W_t, s_{bid}, a_t\right)\right] &\geq \mathbb{E}_0^{\mathbb{P}_W}\left[\sum_{t=0}^{T} D_t(W_t) * U\left(W_t, 0, 0\right)\right] \\
\mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}\left[-\sum_{t=0}^{T} \mathrm{e}^{-\rho t-\lambda*(W_t-W_{t-1})t}\,\mathrm{e}^{-\lambda(W_t-s_{bid}+a_t)}\right] &\geq \mathbb{E}_0^{\mathbb{P}_W}\left[-\sum_{t=0}^{T} \mathrm{e}^{-\rho t-\lambda*(W_t-W_{t-1})t}\,\mathrm{e}^{-\lambda W_t}\right] \\
-\mathrm{e}^{\lambda s_{bid}}\sum_{t=0}^{T}\mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}\left[\mathrm{e}^{-\rho t-\lambda*(W_t-W_{t-1})t-\lambda(W_t+a_t)}\right] &\geq -\sum_{t=0}^{T}\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\rho t-\lambda*(W_t-W_{t-1})t-\lambda W_t}\right]
\end{aligned}
$$

so that

$$
\begin{aligned}
\mathrm{e}^{\lambda s_{bid}} &\leq \frac{\sum_{t=0}^{T}\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\rho t-\lambda*(W_t-W_{t-1})t-\lambda W_t}\right]}{\sum_{t=0}^{T}\mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}\left[\mathrm{e}^{-\rho t-\lambda*(W_t-W_{t-1})t-\lambda(W_t+a_t)}\right]} \\
\mathrm{e}^{\lambda s_{bid}} &\leq \frac{\sum_{t=0}^{T}\mathrm{e}^{-\rho t}\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda*(W_t-W_{t-1})t-\lambda W_t}\right]}{\sum_{t=0}^{T}\mathrm{e}^{-\rho t}\mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}\left[\mathrm{e}^{-\lambda*(W_t-W_{t-1})t-\lambda(W_t+a_t)}\right]} \\
s_{bid} &\leq \frac{1}{\lambda}\left(\ln\sum_{t=0}^{T}\mathrm{e}^{-\rho t}\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda W_t(1+t)+\lambda W_{t-1}t}\right]-\ln\sum_{t=0}^{T}\mathrm{e}^{-\rho t}\mathbb{E}_0^{\mathbb{P}_W \times \mathbb{P}_a}\left[\mathrm{e}^{-\lambda W_t(1+t)+\lambda W_{t-1}t-\lambda a_t}\right]\right),
\end{aligned}
$$

which gives the maximum price the buyer is willing to pay for the swap. □

A.2. **Proof of proposition 4.1.**

Proof.

$$
\begin{aligned}
&\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\Lambda_t}\right] \\
=\;&\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda\left[(1+t)\epsilon_t+t\sum_{i=1}^{t-1}\beta^i\epsilon_{t-i}\right]}\right] \\
=\;&\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda\left[(1+t)\left(\sigma\xi_t+\sigma\sum_{j=0}^{m}\eta_j\xi_{t-j}^a\right)+t\sum_{i=1}^{t-1}\beta^i\left(\sigma\xi_{t-i}+\sigma\sum_{j=0}^{m}\eta_j\xi_{t-i-j}^a\right)\right]}\right] \\
=\;&\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t-\lambda(1+t)\sigma\sum_{j=0}^{m}\eta_j\xi_{t-j}^a-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^{m}\eta_j\xi_{t-i-j}^a}\right] \\
=\;&\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}}\right],\quad \text{since } \eta_j=0,\forall j \\
=\;&\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t}\right]\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}}\right] \\
=\;&\mathrm{e}^{-\frac{\lambda^2(1+t)^2\sigma^2}{2}}\mathrm{e}^{-\frac{\lambda^2t^2\left(\sum_{i=1}^{t-1}\beta^i\right)^2\sigma^2}{2}} \\
=\;&\mathrm{e}^{-\frac{\lambda^2}{2}\sigma^2\left[(1+t)^2+t^2\left(\sum_{i=1}^{t-1}\beta^i\right)^2\right]},
\end{aligned}
$$

where we have used that

$$
\mathbb{E}\left[X^c\right]=\mathrm{e}^{c\mu+\frac{c^2\sigma^2}{2}},
$$

for X log-normal, and

$$
\xi,\tilde{\xi},\xi^a\sim\mathcal{N}\left(0,1\right).
$$

□

Proof.

$$
\begin{aligned}
& \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\Lambda_t^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda\left[(1+t)\epsilon_t+t\sum_{i=1}^{t-1}\beta^i\epsilon_{t-i}+\epsilon_t^a\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda\left[(1+t)\left(\sigma\xi_t+\sigma\sum_{j=0}^{m}\eta_j\xi_{t-j}^a\right)+t\sum_{i=1}^{t-1}\beta^i\left(\sigma\xi_{t-i}+\sigma\sum_{j=0}^{m}\eta_j\xi_{t-i-j}^a\right)+\sum_{i=1}^{k}\varsigma_t\epsilon_{t-i}^a+\sigma^a\xi_t^a\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t-\lambda(1+t)\sigma\sum_{j=0}^{m}\eta_j\xi_{t-j}^a-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^{m}\eta_j\xi_{t-i-j}^a-\lambda\sum_{i=1}^{k}\varsigma_t\epsilon_{t-i}^a-\lambda\sigma^a\xi_t^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}-\lambda\sum_{i=1}^{k}\varsigma_t\epsilon_{t-i}^a-\lambda\sigma^a\xi_t^a}\right], \quad \text{since } \eta_j=0, \forall j \\
= \; & \mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t}\right]\mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}}\right]\mathbb{E}_0^{\mathbb{P}_a}\left[\mathrm{e}^{-\lambda\sum_{i=1}^{k}\varsigma_t\epsilon_{t-i}^a}\right]\mathbb{E}_0^{\mathbb{P}_a}\left[\mathrm{e}^{-\lambda\sigma^a\xi_t^a}\right] \\
= \; & \mathrm{e}^{-\frac{\lambda^2(1+t)^2\sigma^2}{2}}\mathrm{e}^{-\frac{\lambda^2t^2\left(\sum_{i=1}^{t-1}\beta^i\right)^2\sigma^2}{2}}\mathrm{e}^{-\frac{\lambda^2k\varsigma_t^2}{2}}\mathrm{e}^{-\frac{\lambda^2(\sigma^a)^2}{2}} \\
= \; & \mathrm{e}^{-\frac{\lambda^2}{2}\left[\sigma^2\left[(1+t)^2+t^2\left(\sum_{i=1}^{t-1}\beta^i\right)^2\right]+k\varsigma_t^2+(\sigma^a)^2\right]}.
\end{aligned}
$$

□

Proof.

$$
\begin{aligned}
& \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\Lambda}_t}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\lambda}\left[(1+t)\tilde{\epsilon}_t+t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\epsilon}_{t-i}\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\lambda}\left[(1+t)\left(\tilde{\sigma}\tilde{\xi}_t+\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_i\xi_{t-j}^a\right)+t\sum_{i=1}^{t-1}\tilde{\beta}^i\left(\tilde{\sigma}\tilde{\xi}_{t-i}+\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-i-j}^a\right)\right]}\right] \\
= \; & \mathrm{e}^{-\frac{\tilde{\lambda}^2(1+t)^2\tilde{\sigma}^2}{2}}\mathrm{e}^{-\frac{\tilde{\lambda}^2t^2\left(\sum_{i=1}^{t-1}\tilde{\beta}^i\right)^2\tilde{\sigma}^2}{2}} \\
= \; & \mathrm{e}^{-\frac{\tilde{\lambda}^2}{2}\tilde{\sigma}^2\left[(1+t)^2+t^2\left(\sum_{i=1}^{t-1}\tilde{\beta}^i\right)^2\right]}.
\end{aligned}
$$

□

Proof.

$$
\begin{aligned}
& \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\Lambda}_t^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\lambda}\left[(1+t)\tilde{\epsilon}_t+t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\epsilon}_{t-i}-\tilde{\epsilon}_t^a\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}\times\mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\lambda}\left[(1+t)\left(\tilde{\sigma}\tilde{\xi}_t+\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-j}^a\right)+t\sum_{i=1}^{t-1}\tilde{\beta}^i\left(\tilde{\sigma}\tilde{\xi}_{t-i}+\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-i-j}^a\right)-\sum_{i=1}^{k}\varsigma_t\epsilon_{t-i}^a-\sigma^a\xi_t^a\right]}\right] \\
= \; & \mathrm{e}^{-\frac{\tilde{\lambda}^2(1+t)^2\tilde{\sigma}^2}{2}}\mathrm{e}^{-\frac{\tilde{\lambda}^2t^2\left(\sum_{i=1}^{t-1}\tilde{\beta}^i\right)^2\tilde{\sigma}^2}{2}}\mathrm{e}^{-\frac{\tilde{\lambda}^2k\varsigma_t^2}{2}}\mathrm{e}^{-\frac{\tilde{\lambda}^2(\sigma^a)^2}{2}} \\
= \; & \mathrm{e}^{-\frac{\tilde{\lambda}^2}{2}\left[\tilde{\sigma}^2\left[(1+t)^2+t^2\left(\sum_{i=1}^{t-1}\tilde{\beta}^i\right)^2\right]+k\varsigma_t^2+(\sigma^a)^2\right]}.
\end{aligned}
$$

□

A.3. **Proof of proposition 4.2.**

Proof.

$$
\begin{aligned}
& \mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\Lambda_t}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda\left[(1+t)\epsilon_t+t\sum_{i=1}^{t-1}\beta^i\epsilon_{t-i}\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda\left[(1+t)\left(\sigma\xi_t+\sigma\sum_{j=0}^m\eta_j\xi_{t-j}^a\right)+t\sum_{i=1}^{t-1}\beta^i\left(\sigma\xi_{t-i}+\sigma\sum_{j=0}^m\eta_j\xi_{t-i-j}^a\right)\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t-\lambda(1+t)\sigma\sum_{j=0}^m\eta_j\xi_{t-j}^a-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^m\eta_j\xi_{t-i-j}^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t}\mathrm{e}^{-\lambda(1+t)\sigma\sum_{j=0}^m\eta_j\xi_{t-j}^a}\mathrm{e}^{-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}}\mathrm{e}^{-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^m\eta_j\xi_{t-i-j}^a}\right] \\
= \; & \mathrm{e}^{\frac{1}{2}\left[-\lambda(1+t)\sigma-\lambda(1+t)\sigma\sum_{j=0}^m\eta_j-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^m\eta_j\right]^2}.
\end{aligned}
$$

where we have used that $\prod_{i=1}^n X_i \sim \log\mathcal{N}\left(n\mu,\sum_{i=1}^n\sigma_i^2\right)$, for $X_i \sim \log\mathcal{N}\left(\mu,\sigma_i^2\right)$, $i = 1,\ldots,n$, independent log-normally distributed variables with the same mean parameter μ and possibly varying volatility σ_i. In this case, $\prod_{i=1}^n X_i \sim \log\mathcal{N}(0,1)$, as $X_i \sim \log\mathcal{N}(0,1)$, $i = 1,\ldots,n$; thus, $\mathbb{E}\left[X^c\right] = \mathrm{e}^{\frac{c^2}{2}}$. □

Proof.

$$
\begin{aligned}
& \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\Lambda_t^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda\left[(1+t)\epsilon_t+t\sum_{i=1}^{t-1}\beta^i\epsilon_{t-i}+\epsilon_t^a\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda\left[(1+t)\left(\sigma\xi_t+\sigma\sum_{j=0}^m\eta_j\xi_{t-j}^a\right)+t\sum_{i=1}^{t-1}\beta^i\left(\sigma\xi_{t-i}+\sigma\sum_{j=0}^m\eta_j\xi_{t-i-j}^a\right)+\sum_{i=1}^k\varsigma_t\epsilon_{t-i}^a+\sigma^a\xi_t^a\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t-\lambda(1+t)\sigma\sum_{j=0}^m\eta_j\xi_{t-j}^a-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^m\eta_j\xi_{t-i-j}^a-\lambda\sum_{i=1}^k\varsigma_t\epsilon_{t-i}^a-\lambda\sigma^a\xi_t^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_W\times\mathbb{P}_a}\left[\mathrm{e}^{-\lambda(1+t)\sigma\xi_t}\mathrm{e}^{-\lambda(1+t)\sigma\sum_{j=0}^m\eta_j\xi_{t-j}^a}\mathrm{e}^{-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\xi_{t-i}}\mathrm{e}^{-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^m\eta_j\xi_{t-i-j}^a}\mathrm{e}^{-\lambda\sum_{i=1}^k\varsigma_t\epsilon_{t-i}^a}\mathrm{e}^{-\lambda\sigma^a\xi_t^a}\right] \\
= \; & \mathrm{e}^{\frac{1}{2}\left[-\lambda(1+t)\sigma-\lambda(1+t)\sigma\sum_{j=0}^m\eta_j-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma-\lambda t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^m\eta_j-\lambda\sum_{i=1}^k\varsigma_t-\lambda\sigma^a\right]^2}.
\end{aligned}
$$

□

Proof.

$$
\begin{aligned}
& \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\Lambda}_t}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\lambda}\left[(1+t)\tilde{\epsilon}_t+t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\epsilon}_{t-i}\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\lambda}\left[(1+t)\left(\tilde{\sigma}\tilde{\xi}_t+\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_i\xi_{t-j}^a\right)+t\sum_{i=1}^{t-1}\tilde{\beta}^i\left(\tilde{\sigma}\tilde{\xi}_{t-i}+\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_j\xi_{t-i-j}^a\right)\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\lambda}(1+t)\tilde{\sigma}\tilde{\xi}_t-\tilde{\lambda}(1+t)\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_i\xi_{t-j}^a-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\tilde{\xi}_{t-i}-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_j\xi_{t-i-j}^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}}}\left[\mathrm{e}^{-\tilde{\lambda}(1+t)\tilde{\sigma}\tilde{\xi}_t}\mathrm{e}^{-\tilde{\lambda}(1+t)\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_i\xi_{t-j}^a}\mathrm{e}^{-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\tilde{\xi}_{t-i}}\mathrm{e}^{-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_j\xi_{t-i-j}^a}\right] \\
= \; & \mathrm{e}^{\frac{1}{2}\left[-\tilde{\lambda}(1+t)\tilde{\sigma}-\tilde{\lambda}(1+t)\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_j-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\sum_{j=0}^m\tilde{\eta}_j\right]^2}.
\end{aligned}
$$

□

Proof.

$$
\begin{aligned}
& \mathbb{E}_0^{\mathbb{P}_{\tilde{W}} \times \mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\Lambda}_t^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}} \times \mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\lambda}\left[(1+t)\tilde{\epsilon}_t+t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\epsilon}_{t-i}-\tilde{\epsilon}_t^a\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}} \times \mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\lambda}\left[(1+t)\left(\tilde{\sigma}\tilde{\xi}_t+\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-j}^a\right)+t\sum_{i=1}^{t-1}\tilde{\beta}^i\left(\tilde{\sigma}\tilde{\xi}_{t-i}+\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-i-j}^a\right)-\sum_{i=1}^{k}\varsigma_t\epsilon_{t-i}^a-\sigma^a\xi_t^a\right]}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}} \times \mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\lambda}(1+t)\tilde{\sigma}\tilde{\xi}_t-\tilde{\lambda}(1+t)\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-j}^a-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\tilde{\xi}_{t-i}-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-i-j}^a+\tilde{\lambda}\sum_{i=1}^{k}\varsigma_t\epsilon_{t-i}^a+\tilde{\lambda}\sigma^a\xi_t^a}\right] \\
= \; & \mathbb{E}_0^{\mathbb{P}_{\tilde{W}} \times \mathbb{P}_a}\left[\mathrm{e}^{-\tilde{\lambda}(1+t)\tilde{\sigma}\tilde{\xi}_t}\mathrm{e}^{-\tilde{\lambda}(1+t)\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-j}^a}\mathrm{e}^{-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\tilde{\xi}_{t-i}}\mathrm{e}^{-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j\xi_{t-i-j}^a}\mathrm{e}^{\tilde{\lambda}\sum_{i=1}^{k}\varsigma_t\epsilon_{t-i}^a}\mathrm{e}^{\tilde{\lambda}\sigma^a\xi_t^a}\right] \\
= \; & \mathrm{e}^{\frac{1}{2}\left[-\tilde{\lambda}(1+t)\tilde{\sigma}-\tilde{\lambda}(1+t)\tilde{\sigma}\sum_{j=0}^{m}\tilde{\eta}_j-\tilde{\lambda}t\sum_{i=1}^{t-1}\tilde{\beta}^i\tilde{\sigma}-\tilde{\lambda}t\sum_{i=1}^{t-1}\beta^i\sigma\sum_{j=0}^{m}\eta_j+\tilde{\lambda}\sum_{i=1}^{k}\varsigma_t+\tilde{\lambda}\sigma^a\right]^2}.
\end{aligned}
$$

□

Zürcher Kantonalbank
E-mail address: miret.padovani@zkb.ch

Zürcher Kantonalbank and Swiss Finance Institute
E-mail address: paolo.vanini@zkb.ch

SOLIDARITY GROUP LENDING AND GLOBAL GAMES

MIRET PADOVANI

ABSTRACT. I study microfinance solidarity group lending within a global game framework. Group members invest independently in either a safe or a risky project. *Ex ante* observation and coordination on the choice of projects is not always possible. Though risky projects allow microentrepreneurs to extract risk premia, safe projects raise the probability of success of the group loan, in which case group members benefit from access to business support services provided by the microfinance institution. I show how borrowers' perception of the strength of social cohesion within the microentrepreneurial community induces coordination on safe project choices.

1. INTRODUCTION

Solidarity group lending has been a pillar of microfinance ever since its introduction in Bangladesh in the seventies by Muhammad Yunus, founder of the Grameen Bank. Roughly half of the 480 microfinance institutions (MFIs) covered by the 2007 MixMarket MFI Benchmarks dataset[1] use a mix of solidarity and individual lending, while 27 MFIs lend exclusively through the solidarity group method.[2]

The critical feature of solidarity group loans is the joint liability of all group members. The seminal group lending scheme is that of the Grameen Bank. This scheme has been adopted by other MFIs, though sometimes with a couple of differing features. In a Grameen-style loan a candidate borrower must form a group with other four people. In what is known as the 2:2:1 staggering scheme, two members of the group originally receive a loan, and if they do well, other two then receive loans, and, finally, so does the group chairperson. The borrowers are encouraged to assist each other, and all loan disbursements and repayments are made publicly at a weekly meeting. Loans are typically given out for one year, at a relatively high fixed interest rate, and are always for modest amounts: no more than a few hundred dollars. The borrowers use the loans to make small capital investments,

---

Date: First version: January 2010. This version: February 18, 2010.

Key words and phrases. Solidarity group lending; microfinance; global games; social cohesion.

I am highly indebted to Hyun Song Shin for many discussions and helpful comments throughout the writing of this paper. Special thanks also to Paolo Vanini for invaluable comments on an earlier draft of this paper.

[1]See www.mixmarket.org.

[2]Solidarity group lending needs to be distinguished from village banking, which I do not consider in this paper. There are a few major differences between the two lending schemes. First of all, village banking does not apply to self-formed groups; rather, individuals typically arrive on their own and are placed on a waiting list until a desired number of borrowers is reached. Second, village banking groups are composed of a much higher number of members: up to 40 rather than 5 or 6. A third and crucial difference is that village banking does not impose joint liability among members, although members are encouraged to supervise each other, e.g. by visiting a person in default and inquiring as to the cause of default or late repayment.

e.g. for the production of bakery goods. Once all group members have repaid the initial loan, they are allowed to apply for new ones.

As microentrepreneurs typically lack physical collateral which they can pledge against the loans they receive, MFIs need to make use of collateral substitutes. The MFI can in most settings exploit the social sanction opportunities that exist between borrowers. Social collateral generated with group formation and imposition of joint responsibility at the group level may be deemed as an effective substitute for conventional creditworthiness. As such, social collateral offers a guarantee for loan repayment and minimizes potential loan defaults.

Social sanctions may take several forms. Defecting borrowers may be excluded from privileged access to input supplies, from further trade credit, from social and religious events, or from day-to-day courtesies. The threat of such sanctions is more credible the smaller the village community.[3] Indeed, the strength of social sanctions mirrors the strength of social cohesion, i.e. the network of social relationships binding people together in communities and neighborhoods.

It is hard to give a precise definition of social collateral, given its multiple facets. Nevertheless, it is worthy to note at this stage the distinction made in the sociology literature between social capital and social cohesion, both of which constitute a type of social collateral:

> "Social capital is a characteristic of the individual, defined as the current sacrifice in time, effort, or money made by that individual in the hope of promoting productive cooperation or coordination with others (Osberg [Osb03])."

On the other hand,

> "Social cohesion is a characteristic of the society or collective that depends on the history of the accumulation of social capital in the group and which, in turn, affects the incentives for current social capital investments (Osberg [Osb03])."

Both aspects of social collateral are crucial for the success of microfinance: social capital determines the creditworthiness of individual borrowers, social cohesion that of an entire group of borrowers. My interest in this paper focusses on the latter.[4]

The importance of social collateral in financial transactions is not confined to microfinance, but also applies to the more general case of financial transactions among friends and acquaintances: interest rates may be low or even zero, but social costs and obligations are often considerable.

Empirical studies give, however, a mixed picture concerning the extent to which social cohesion affects the performance of group lending. Differences in results sometimes seem to depend on the proxy used. Wydick [Wyd99] shows that groups of strangers can do as well as groups of friends - if not even better. Social ties *per se* have little impact on the performance of borrowing groups. What really matter are peer monitoring and group pressure. Peer monitoring denotes the ability of borrowers to monitor the investment behavior of another during the course of the loan, making sure that each group member undertakes only safe investment projects with borrowed capital; whereas, group pressure denotes the ability to threaten

[3]See discussion in Armendariz and Morduch [AM05].

[4]Osberg [Osb03] analyzes the economic implications of social cohesion. This field of research is concerned with the question of whether differences in trust, social capital, social cohesion can explain differences in the economic performance of world regions. See also Dayton-Johnson [DJ01].

exclusion of non-paying members from continued access to credit. Since friends may be softer and more forgiving on each other, it is not necessarily true that stronger social ties lead to higher repayment rates.

On the other hand, subsequent field experiments conducted in several developing countries by the same author,[5] as well as research by Gomez and Santor [GS03] demonstrate that the strength and credibility of the threat of social sanctions against defaulting members is the key determinant in group loan performance.

While the inspiration for my work lies in the literature on the role of social sanctions in group-based lending schemes,[6] the modeling framework is related to the literature on coordination in global games. Global games are games with incomplete information regarding both the fundamental state of the world and the actions taken by other players. They have first been studied by Carlsson and van Damme [CvD93] and later on extended by Morris and Shin [MS03] to study financial crises and bank runs. The question I ultimately want to answer is whether private beliefs concerning the strength of social cohesion within the microentrepreneurial community induce group loan members to act in the interest of the entire group rather than in their own self-interest.

To the best of my knowledge, there has been only one application of global games to microfinance lending by Bond and Rai [BR09]. Although their theoretical idea is the same as mine, i.e. the authors study a borrower's motivation to pay back in a solidarity group loan within a global game framework, there are at least two substantial differences. First of all, in their game, players choose between paying back a loan or not, rather than between investing in a safe or a risky project. So also the definition of the game's payoffs and the subsequent analysis differ substantially. More crucially, the motivation behind Bond and Rai's [BR09] paper is different than mine: their focus lies in finding the optimal contract from an MFI's perspective, whereas my focus lies in studying the role of social cohesion in a global games framework.

The remainder of the paper is organized as follows. The next section describes joint liability microfinance games - in the setting they are most typically carried out in laboratory experiments. The following section outlines the benchmark global game modeling framework for a 2-player game, where each microentrepreneur takes an investment decision under uncertainty regarding the action of the other group member ('strategic uncertainty'). Sections 4 to 5 extend the benchmark model to 3-player and $(n+1)$-player games, respectively. Section 6 introduces uncertainty regarding the strength of social cohesion ('fundamental uncertainty'), thus leading to a 'private values' global game. Sections 7 to 8 apply the 'private values' model to 3-player and $(n+1)$-player games, respectively. Section 9 discusses the economics behind my results. Section 10 concludes.

2. Joint liability microfinance games

Several researchers have undertaken laboratory experiments, where they simulate microfinance games in an attempt to identify the key factors to higher repayment rates. Despite the disadvantage of proceeding in a deliberately artificial setting, lab experiments have the advantage of being able to keep all features of the game constant from one round to the other, while only varying the parameters that

[5]See Cassar *et al.* [CCW07], Cassar and Wydick [CW09], and Wydick *et al.* [WHK09].
[6]See Karlan [Kar07], Karlan *et al.* [KMRS09], and Ambrus *et al.* [AMS08].

are to be tested for. For the purposes of my study, I outline microfinance games similar to the joint liability experiments conducted by Gine *et al.* [GJKM06] in the Philippines. The games are inspired by Stiglitz' [Sti90] model of *ex ante* moral hazard in microfinance and are designed to capture the most relevant features of microfinance loans, namely peer insurance and social costs to individual default. Although Gine *et al.* [GJKM06] consider two-player partnerships, their discussion can be extended to n-player partnerships.

Consider a joint liability group composed of n borrowers. Each group member is given a loan with face value L and can undertake one of two projects: either a safe project, yielding Y_S for sure (i.e. with probability of success $\pi_S = 1$), or a risky project, yielding $Y_R > Y_S$ with probability of success $\pi_R < 1$ and 0 otherwise. A borrower has to repay her loan only if her project is successful; she may not use wealth from prior investments to repay the current investment's loan.

The success of borrowers' projects are independent. If all partners choose safe investments, then the outcome of the game is identical to that of an individual liability game: Each borrower receives Y_S and pays back her loan; no borrower has to compensate the MFI for any defaulting group member.

Another possible situation is when safe investors are matched with risk-taking borrowers; then the safe investors can be expected to bail out their partners. Each safe investor receives Y_S, but in addition to repaying her loan, she must also compensate the MFI for the shortfall due to any defaulting risk-taking member. Successful risk-taking borrowers, too, must contribute in bailing out their defaulting peers. Group default is more likely to occur if too many investors invest in risky projects. This is because the likelihood of project failures rises and so does the likelihood that not enough borrowers can compensate the MFI for too big a shortfall.

Borrowers do not communicate their investment choices and *ex ante* observation of investment choices is not possible. This assumption begs the question of whether ex ante observation is truly never possible in microfinance investments, or only sometimes. Though the nature of the business may very well be observable (e.g. producing scarves), the suppliers, for example, may not be. So supplying to a customer known to be risky, e.g. because paying in an unstable or weak currency, or having poor businesses themselves, or likely to 'run away with the money', can be interpreted as undertaking a risky investment. W.l.o.g. assume *ex post* observation of risky or safe investment choices occurs at the time when the loan is due back to the MFI. If it is revealed that individual j had engaged in risky transactions, then the other group members and/or society can impose on j a social sanction c, which represents the (common) cost of deviating from the social optimum.

Three major results of Gine *et al.* [GJKM06] are particularly worthy of note. First of all, adding an opportunity for *ex post* punishment in the dynamic monitoring game does not alter the average repayment rate. So the standard dynamic game results seem not to be that useful in explaining the high repayment rates in solidarity group lending. Second, endogenous partner choice has a big effect on contract performance. This is a key property suggesting that social sanctions play an important role. A third result is that the best predictor of partner choice is having been a host or guest of the partner in real life. Together with the previous result, this suggests that coordination is better achieved when playing with someone the microentrepreneur knows well, and against whom she has other social relationships.

Remark 2.1 (On expected returns). In Stiglitz' [Sti90] original model, the safe(r) project is assumed to deliver a higher expected return than the riskier project, i.e.

$$Y_S - (1+r)L \geq \pi_R \left[Y_R - (1+r)L\right].$$

But this assumption is implausible. An investor who desires a higher expected return on her investments should be willing to undertake higher risk. *Vice versa*, risk-averse investors need to be rewarded for undertaking undiversifiable, systematic risk. The expected return on a risky investment is then equal to the sure return on a risk-free investment plus a risk premium.

3. A 2-borrower group example

I analyze the joint liability microfinance game in a global game setting. Each borrower faces strategic uncertainty, i.e. uncertainty over the actions of the other players. More specifically, strategic uncertainty is the uncertainty over how many group members will undertake risky or safe projects. From the perspective of borrower i, a critical number k^* of her peers need to invest their funds safely, so that the group loan is successful. In case of success of the group loan, all group members have access to MFI business support services and are entitled to apply for new loans in the future, if they decide to do so. To start with, consider the case where social cost c is common knowledge among all borrowers. So there is no fundamental uncertainty, which I will instead introduce at a later stage. Also start with 2 borrowers, i and j.

As I will illustrate throughout the next sections, the incentive of choosing a safe project for borrower i is increasing in the number of her peers also choosing the safe investment: with a lower number of possibly defaulting risky projects, a safe-investing borrower faces a lower risk of having to bail out her peers and therefore has a higher expected payoff on her investment. Thus there are strategic complementarities between microentrepreneurs' investment choices.

The only non-trivial case in the two-player game is when both investors need to invest in a safe project for the group loan to be successful, otherwise the game gives rise to a free-riding problem. The payoff matrix for $k^* = 2$ is given in table 3.1 with the following notation:

$$(A;\alpha) = (\Pi_{i,S};\Pi_{j,S}) = \left(Y_S - (1+r)L + \overline{Z};\ Y_S - (1+r)L + \overline{Z}\right);$$

$$(B;\beta) = (\Pi_{i,R};\Pi_{j,S}) = (\pi_{i,R}\ (Y_R - (1+r)L) - c;\ Y_S - (1+r)L);$$

$$(C;\gamma) = (\Pi_{i,S};\Pi_{j,R}) = (Y_S - (1+r)L;\ \pi_{j,R}\ (Y_R - (1+r)L) - c);$$

$$(D;\delta) = (\Pi_{i,R};\Pi_{j,R}) = (\pi_{i,R}\ (Y_R - (1+r)L) - c;\ \pi_{j,R}\ (Y_R - (1+r)L) - c).$$

i; j	S	R
S	$(A;\alpha)$	$(C;\gamma)$
R	$(B;\beta)$	$(D;\delta)$

TABLE 3.1. Two-action, two-player game payoff matrix

The term Z denotes the payoff a borrower gets from the group loan being successful. That is, Z is the value a microentrepreneur assigns to the business services the MFI provides its clients with. We may also think of Z more broadly: Z would then include both monetary benefits, such as future loan amounts, as well as non-monetary benefits, such as having access to business services provided by the MFI, not going through the hassle of looking for a new group, being confident enough that there will be continuous funding for the started investment, and so on. The parameter Z can actually take on two values, according to whether the borrower invests safely or not: a higher-valued $\overline{Z}$ and a lower-valued $\underline{Z}$. Good borrowers can expect to extract the maximum benefits from the MFI's services; hence, $\overline{Z}$. Whereas, borrowers with an imperfect credit record have limited access to an MFI's services, so that a deduction in offered services, i.e. $\overline{Z} - \underline{Z}$, can be interpreted as a 'sanction' from the MFI. Note, however, that in this specific example with two borrowers, only $\overline{Z}$ comes into the picture. It will result from the following discussion that the value Z of the MFI's services influences the behavior of group members and is, therefore, of crucial importance in determining the outcome of the group loan.

Note that this game differs from the prisoners' dilemma, where the players have a dominant action not to cooperate. Here, we have multiple equilibria and the good outcome is also an equilibrium. From the entire group's perspective it is better to invest in safer projects so not to jeopardize the group loan. Riskier projects do enable individual members to enjoy some private benefits in case of success, but lower the verifiable payoff for the group, hence putting the group loan at too high a risk of interruption. The risk premium captures, therefore, the tension between the socially desirable outcome and the outcome resulting from the self-interested action of group members.

I simplify the notation in the remainder of the paper as follows.

- $\mathbb{E}[Y_R] = \pi_R\,(Y_R - (1+r)\,L)$, the expected return on the risky project - after paying back the loan principal plus interest;
- $Y_S = Y_S - (1+r)\,L$, the sure return on the safe project - after paying back the loan principal plus interest;
- $\mathbb{E}[\lambda_x]$, the maximum expected compensation due to the lender if any or all of borrower i's x risk-taking peers default;
- $P = \mathbb{E}[Y_R] - Y_S$, the gross risk premium;
- $p = P - c$, the risk premium net of social sanctions;
- $\overline{Z} - \underline{Z} = z$, the extra benefit from MFI services.

"Both invest in safe projects" is the dominant equilibrium when c is large enough that

$$A > B;\ C > D$$

for borrower i and

$$\alpha > \gamma;\ \beta > \delta$$

for borrower j. That is, when c is such that both

$$Y_S + \overline{Z} > \mathbb{E}[Y_R] - c$$

and

$$Y_S > \mathbb{E}[Y_R] - c$$

hold. So when

$$c > \mathbb{E}[Y_R] - Y_S,$$

i.e. when c is larger than the risk premium, then defecting from the social optimum is never rewarding and it is a dominant strategy for both borrowers to go for the safe project. Put differently: When social sanctions lead to a negative risk premium, i.e.

$$\mathbb{E}[Y_R] - Y_S - c < 0 \Rightarrow p < 0,$$

then both microentrepreneurs invest in safe projects.

"Both invest in risky projects" is the dominant equilibrium when c is small enough that

$$B > A;\ D > C$$

for borrower i and

$$\gamma > \alpha;\ \delta > \beta$$

for borrower j. That is, when c is such that both

$$\mathbb{E}[Y_R] - c > Y_S + \overline{Z}$$

and

$$\mathbb{E}[Y_R] - c > Y_S$$

hold. So when

$$c < \mathbb{E}[Y_R] - Y_S + \overline{Z},$$

i.e. when c is smaller than the difference between the risk premium and the value of MFI services, then cooperating towards the social optimum is never rewarding and it is a dominant strategy for borrower i to go for the risky project. Put differently: When social sanctions do not lower the risk premium below the value of MFI services, i.e.

$$\mathbb{E}[Y_R] - Y_S - c > \overline{Z} \Rightarrow p < \overline{Z},$$

then both microentrepreneurs invest in risky projects.

There are multiple equilibria when it holds that

$$A > B;\ D > C$$

for borrower i and

$$\alpha > \gamma;\ \delta > \beta$$

for borrower j. That is, when c is such that both

$$Y_S + \overline{Z} \geq \mathbb{E}[Y_R] - c$$

and

$$\mathbb{E}[Y_R] - c \geq Y_S$$

hold. Then the risk premium net of social sanctions lies somewhere within the interval $[0, \overline{Z}]$. The equilibria are Pareto-rankable when $A > D$, or

$$Y_S + \overline{Z} \geq \mathbb{E}[Y_R] - c,$$

so that it would be in both microentrepreneurs' interest to coordinate on undertaking safe investments. A coordination failure arises when the Pareto-superior equilibrium fails to be selected.

4. A 3-borrower group example

A transition example from the 2-player game to the $(n+1)$-player game will allow us to further clarify our setting, as the number of group members increases. Let us take a group of three borrowers. Borrower i has $n = 2$ peers, j and l. Assume it is essential that at least two borrowers invest in a safe project for the group loan to be successful. Say borrower i decides to invest in a safe project, then at least $k^* = 1$ of her peers must invest in a safe project, too, if the group loan were to be successful. I will assume throughout the paper that all borrowers' risky projects carry the same probability of success; hence, in this example, $\pi_{i,R} = \pi_{j,R} = \pi_{l,R}$.

The expected payoff for borrower i if investing in a safe project is given by the following.

- Payoff if $k = 2$, i.e. if both her peers invest safely and the group loan is successful, since $k \geq k^*$:
$$\Pi_{i,S} = Y_S + \overline{Z}.$$
- Payoff if $k = 1$, i.e. if also another of her peers invests safely and the group loan is successful, since $k \geq k^*$:
$$\Pi_{i,S} = Y_S - \mathbb{E}[\lambda_1] + \overline{Z}.$$
If the risk-taking borrower is unsuccessful, then borrower i, together with the other safe borrower, needs to compensate the lender for an amount not higher than λ_1. However, since the loan is nevertheless successful, borrower i receives benefit $\overline{Z}$.
- Payoff if $k = 0$, i.e. if no other borrower invests safely and the group loan fails, since $k < k^*$:
$$\Pi_{i,S} = Y_S.$$

The expected payoff for borrower i if investing in a risky project is given by the following.

- Payoff if $k = 2$, i.e. if both her peers invest safely and the group loan is successful, since $k \geq k^*$:
$$\Pi_{i,R} = \mathbb{E}[Y_R] - c + \underline{Z}.$$
- Payoff if $k = 1$, i.e. if only one of her peers invests safely and the group loan fails, since $k < k^*$:
$$\Pi_{i,R} = \mathbb{E}[Y_R] - c.$$
- Payoff if $k = 0$, i.e. if none of her peers invest safely and the group loan fails, since $k < k^*$:
$$\Pi_{i,R} = \mathbb{E}[Y_R] - c.$$

We are assuming that a borrower investing in a risky project faces social sanctions even if her project proves out to be successful. It will, indeed, be revealed to other villagers that she had acted egoistically and she would be considered from thereon an untrustworthy business partner.

See the summarizing payoff matrix 4.1 where I make use of the following notation:

- $A = \mathbb{E}[Y_R] - c + \underline{Z}$
- $B = \mathbb{E}[Y_R] - c$
- $C = Y_S + \overline{Z}$
- $D = Y_S - \mathbb{E}[\lambda_1] + \overline{Z}$

	$(i;\, j;\, l)$
SSS	(C; C; C)
SRS	(D; A; D)
SSR	(D; D; A)
SRR	(E; B; B)
RSS	(A; D; D)
RRS	(B; B; E)
RSR	(B; E; B)
RRR	(B; B; B)

TABLE 4.1. Two-action, three-player game payoffs

- $E = Y_S$

Section A in the appendix reports detailed payoffs.

"All invest in safe projects" is the dominant equilibrium when c is large enough such that

$$C > A;\ D > B;\ E > B$$

hold for each borrower. That is, when c is such that

$$Y_S + \overline{Z} > \mathbb{E}\left[Y_R\right] - c + \underline{Z},$$

$$Y_S - \mathbb{E}\left[\lambda_1\right] + \overline{Z} > \mathbb{E}\left[Y_R\right] - c,$$

and

$$Y_S > \mathbb{E}\left[Y_R\right] - c$$

hold. So when

$$c > \mathbb{E}\left[Y_R\right] - Y_S \ \Rightarrow\ p < 0,$$

i.e. when the risk premium net of social sanctions is negative, defecting from the social optimum is never rewarding and it is a dominant strategy for all borrowers to go for the safe project.

"All invest in risky projects" is the dominant equilibrium when c is small enough such that

$$A > C;\ B > D;\ B > E$$

hold for each borrower. That is, when c is such that

$$\mathbb{E}\left[Y_R\right] - c + \underline{Z} > Y_S + \overline{Z},$$

$$\mathbb{E}\left[Y_R\right] - c > Y_S - \mathbb{E}\left[\lambda_1\right] + \overline{Z},$$

and

$$\mathbb{E}\left[Y_R\right] - c > Y_S - (1+r)L$$

hold. So when

$$c < \mathbb{E}\left[Y_R\right] - \left[Y_S - \mathbb{E}\left[\lambda_1\right] + \overline{Z}\right] \ \Rightarrow\ p > \overline{Z} - \mathbb{E}\left[\lambda_1\right]$$

i.e. when the risk premium net of social sanctions is larger than the difference between the value of MFI services and the expected compensation due to the MFI in case of peers' default, cooperating towards the social optimum is never rewarding and it is a dominant strategy for all borrowers to go for the risky project.

There are multiple equilibria when it holds that

$$C > A;\ D > B;\ B > E$$

	$(i;\, j;\, l)$
SSS	($\underline{\mathbf{C}}$; $\underline{\mathbf{C}}$; $\underline{\mathbf{C}}$)
SRS	($\underline{\mathbf{D}}$; A; $\underline{\mathbf{D}}$)
SSR	($\underline{\mathbf{D}}$; $\underline{\mathbf{D}}$; A)
SRR	(E; B; B)
RSS	(A; $\underline{\mathbf{D}}$; $\underline{\mathbf{D}}$)
RRS	(B; B; E)
RSR	(B; E; B)
RRR	($\underline{\mathbf{B}}$; $\underline{\mathbf{B}}$; $\underline{\mathbf{B}}$)

TABLE 4.2. Multiple equilibria in the two-action, three-player game payoffs: SSS and RRR.

for all borrowers. That is, when c is such that

$$Y_S + \overline{Z} \geq \mathbb{E}\left[Y_R\right] - c$$

and

$$\mathbb{E}\left[Y_R\right] - c \geq Y_S$$

hold. Then the risk premium net of social sanctions lies somewhere within the interval $[0, z]$. The equilibria are Pareto-rankable when $C > B$, or

$$Y_S + \overline{Z} > \mathbb{E}\left[Y_R\right] - c,$$

so that it would be in both microentrepreneurs' interest to coordinate on safe investments. Once again, a coordination failure arises when the Pareto-superior equilibrium fails to be selected.

5. An $(n+1)$-borrower group example

Having analyzed in detail the 3-borrower case, we may generalize the framework to the case where borrower i takes a loan with n peers. The payoff matrix contains 2^{n+1} entries for all possible combinations.

The payoff for borrower i when opting for a safe project is

$$\Pi_{i,S} = Y_S - \mathbb{E}\left[\lambda_{n-k}\right] + \overline{Z},$$

if $k \geq k^*$, and

$$\Pi_{i,S} = Y_S,$$

if $k < k^*$.

Her payoff when opting for a risky project is

$$\Pi_{i,R} = \mathbb{E}\left[Y_R\right] - \mathbb{E}\left[\lambda_{n-k}\right] - c + \underline{Z},$$

if $k > k^*$, and

$$\Pi_{i,R} = \mathbb{E}\left[Y_R\right] - c,$$

if $k \leq k^*$.

Figure 5.1 illustrates all possible payoffs. The expected payoff (and also the incentive of choosing a safe project) is increasing in the number of other players also choosing the safe investment: with a lower number of possibly defaulting risky projects, a safe-investing borrower faces a lower risk of having to bail out her peers and therefore has a higher expected payoff on her investment. Thus there are strategic complementarities between microentrepreneurs' investment choices.

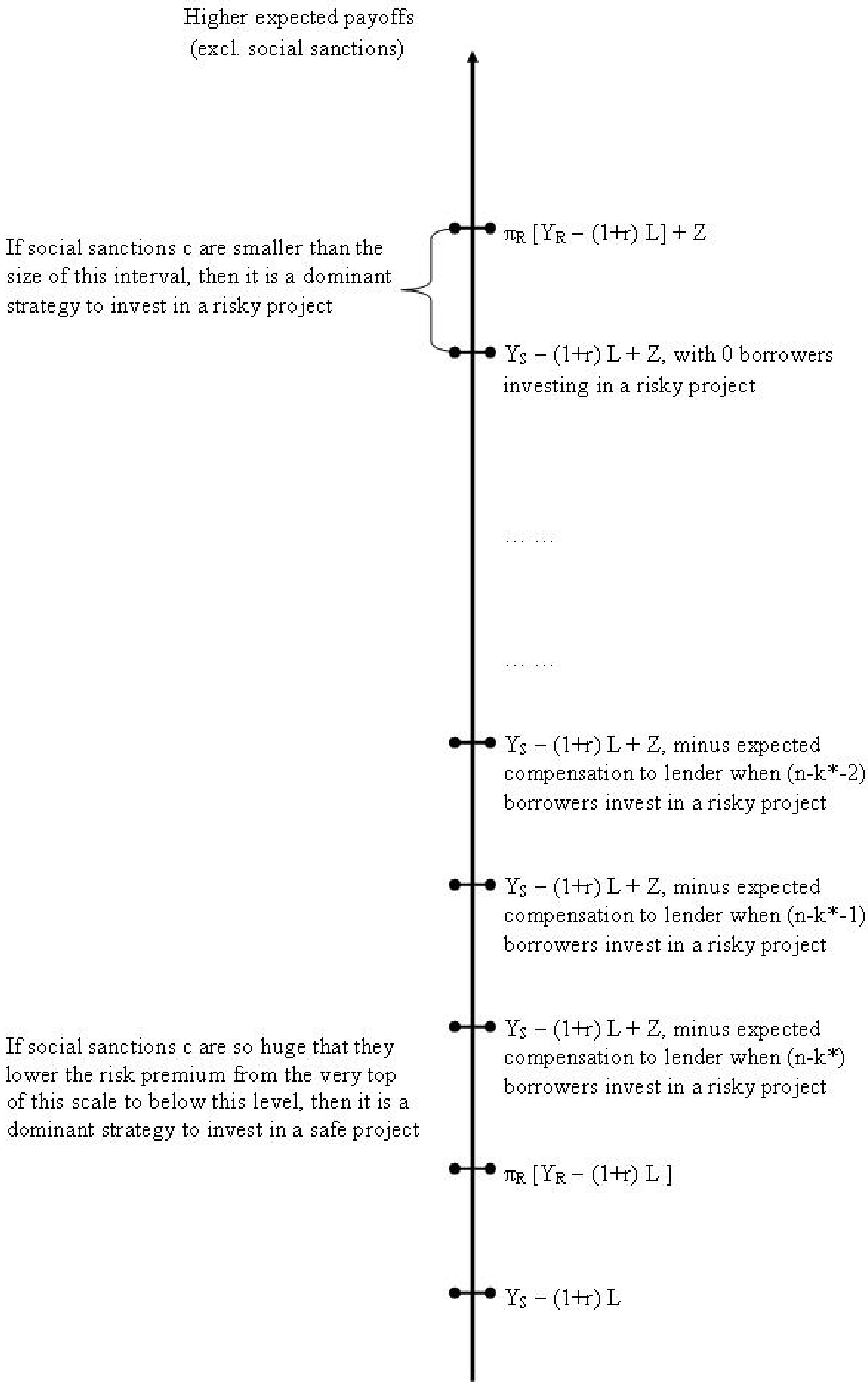

FIGURE 5.1. Visualization of the payoff for borrower i in an $(n+1)$-borrower game.

"All invest in safe projects" is the dominant equilibrium when

$$\begin{aligned}
\mathbb{E}\left[Y_R\right] + \underline{Z} - c &< Y_S + \overline{Z} \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_1\right] - c &< Y_S + \overline{Z} - \mathbb{E}\left[\lambda_1\right] \\
\vdots &< \vdots \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_{n-k^*-2}\right] - c &< Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*-2}\right] \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_{n-k^*-1}\right] - c &< Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*-1}\right] \\
\mathbb{E}\left[Y_R\right] - c &< Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*}\right] \\
\mathbb{E}\left[Y_R\right] - c &< Y_S.
\end{aligned}$$

It is reasonable to believe that an MFI would not impose on a successful borrower to pay a compensation amount λ higher than the value Z of its own services. So the maximal possible compensation amount must satisfy $\mathbb{E}\left[\lambda_{n-k^*}\right] \leq \underline{Z}$; we may take, for simplicity,

$$\mathbb{E}\left[\lambda_{n-k^*}\right] = \underline{Z}. \tag{5.1}$$

Remark 5.1. Under assumption (5.1), borrower i receives the same payoff both in case the group loan is successful with k^* peers (but not herself) investing in a safe project and in case the group loan is unsuccessful. Indeed, $\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_{n-k^*}\right] - c = \mathbb{E}\left[Y_R\right] - c$.

From the payoffs above, we get

$$c > P,$$

or

$$p < 0.$$

So when social sanctions are larger than the gross risk premium, the net risk premium is negative and it is a dominant strategy for all borrowers to invest in a safe project.

Similarly, "All invest in risky projects" is the dominant equilibrium when

$$\begin{aligned}
\mathbb{E}\left[Y_R\right] + \underline{Z} - c &> Y_S + \overline{Z} \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_1\right] - c &> Y_S + \overline{Z} - \mathbb{E}\left[\lambda_1\right] \\
\vdots &> \vdots \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_{n-k^*-2}\right] - c &> Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*-2}\right] \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_{n-k^*-1}\right] - c &> Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*-1}\right] \\
\mathbb{E}\left[Y_R\right] - c &> Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*}\right] \\
\mathbb{E}\left[Y_R\right] - c &> Y_S.
\end{aligned}$$

Therefore,

$$c < P - \overline{Z} + \underline{Z},$$

or

$$p > z.$$

So when social sanctions are smaller than the difference between the gross risk premium and the extra value of MFI services, then the net risk premium is larger than z and it is a dominant strategy for all borrowers to invest in a risky project.

Multiple equilibria arise when borrower i receives a higher payoff if investing in the safe project when exactly k^* or more of her peers do so too, but receives a higher payoff if investing in the risky project when less than k^* of her peers invest in the safe project.

$$\begin{array}{rcl}
\mathbb{E}\left[Y_R\right] + \underline{Z} - c & < & Y_S + \overline{Z} \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_1\right] - c & < & Y_S + \overline{Z} - \mathbb{E}\left[\lambda_1\right] \\
\vdots & > & \vdots \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_{n-k^*-2}\right] - c & < & Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*-2}\right] \\
\mathbb{E}\left[Y_R\right] + \underline{Z} - \mathbb{E}\left[\lambda_{n-k^*-1}\right] - c & < & Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*-1}\right] \\
\mathbb{E}\left[Y_R\right] - c & < & Y_S + \overline{Z} - \mathbb{E}\left[\lambda_{n-k^*}\right] \\
\mathbb{E}\left[Y_R\right] - c & > & Y_S
\end{array}$$

Therefore,

$$P - \overline{Z} + \underline{Z} < c < P;$$

or

$$0 < p < z.$$

So here the net risk premium takes a value between zero and the extra benefit of MFI services and two strict equilibria arise: either all borrowers invest in safe projects or all invest in risky projects.

In case of flexible group lending, the amount λ that a successful microentrepreneur owes for her defaulting partners is optimally determined.[7]

5.1. **Summarizing.** Each microentrepreneur has the choice between investing in a safe project with sure return Y_S or investing in a risky project with expected return $\mathbb{E}\left[Y_R\right]$. Ignoring social sanctions for a moment, the risk premium, $\mathbb{E}\left[Y_R\right] - Y_S$, is smallest when - in case of group loan success - a safe borrower does not have to compensate the lender for the default of any of her peers; it is largest when a safe borrower has to compensate the lender for the default of all $n - k^*$ risk-taking borrowers. Hence, premia are smaller for group loans requiring a larger k^*. This is indicative of a negative relationship between p^* and k^*.

When social sanctions come into the picture, they erode the risk premium and make risky projects less attractive to microentrepreneurs. This highlights the trade-off microentrepreneurs face when taking on a joint-liability loan. On the one side, they are aware of the benefits of risk-taking for their profits as well as for the local economy. On the other side, they are also aware of the social sanctions they face if their peers view them as selfish or otherwise unreliable business partners.

The 2-, 3-, and $(n+1)$-players games have all shown us that, when social cohesion is common knowledge among all group members, there is a potential for multiple equilibria:

- For $c < P - \overline{Z} + \underline{Z}$ (hence, $p > z$), the group loan is interrupted, since social sanctions are so low (read: social cohesion is so weak) that they do not erode even the smallest possible risk premium. Borrowers will find it more profitable to try and extract private benefits rather than invest too safely for the benefit of the group. Defecting from the social optimum is

[7]See Bohle and Ogden [BO09] for a discussion on social sanctions and flexible group lending.

always rewarding and it is a dominant strategy for borrower i to go for the risky project.

- For $c > P$ (hence, $p < 0$), the group loan is kept alive, since social sanctions are so high (read: social cohesion is so strong) that they would even erode the largest possible risk premium. Borrowers do not find it profitable to try and deviate from the social optimum. Defecting from the social optimum is never rewarding and it is a dominant strategy for borrower i to go for the safe project.
- For $P - \overline{Z} + \underline{Z} < c < P$ (hence, $0 < p < z$), the group loan is at risk of being interrupted. For each $p \in (0, z)$, there are two strict equilibria: "All invest in safe projects" and "All invest in risky projects". Though the latter equilibrium allows individual borrowers to extract private benefits, the former is superior from the entire group's perspective. Which equilibrium will emerge depends on the aggregate investment choices of the borrowers. Cooperating towards the socially desirable outcome is rewarding for borrower i if and only if at least k^* of her peers go for the safe project. Hence, there arises a coordination problem among all borrowers where the beliefs of borrower i over the investment choices of her peers play an important role.

6. Private values global game

At this stage I introduce fundamental uncertainty, i.e. uncertainty over the true social cohesion within the community of microentrepreneurs. To capture this uncertainty, assume that the strength of social sanctions, c, is no longer common knowledge to all borrowers, but is instead heterogeneously perceived across the population. This also leads to the risk premium being heterogeneously perceived. Each borrower's perception of the risk premium, p_i, is composed of two terms: a uniformly distributed population-wide term, θ, and an idiosyncratic term, s_i. Hence,

$$p_i = \theta + s_i,$$

where s_i is i.i.d. with uniform distribution over $[-\epsilon, \epsilon]$, for small $\epsilon > 0$. Plus, s_i and s_j are independent for $i \neq j$. Borrower i knows p_i, but cannot distinguish the θ and s_i; so p_i is a noisy signal.

Note that differences in risk premia is a realistic assumption, given that in reality different risky projects will have different probabilities of success.

Definition 6.1 (Strategy)**.** A strategy is a function specifying an action for each possible private signal. Borrower i's investment decision must be conditional on her own risk premium, p_i, rather than on θ. Hence, a strategy is a mapping

$$p_i \mapsto \{\text{Safe}, \text{Risk}\}$$

and the optimal choice of action for borrower i will depend on the probability she attaches to k exceeding the critical threshold k^*.

Definition 6.2 (Equilibrium)**.** On observing her own risk premium, borrower i reasons her way towards the probability density over k. An equilibrium sets in when each borrower invests in the project that is a best response to her beliefs over the number of her peers investing in either a safe or a risky project.

As I will show in the following sections, in a global game setting, if players have only private information, there is a unique equilibrium where each borrower invests in the project that is a best response to a uniform belief over the number k of her peers investing in a safe project. Thus, when faced with some information concerning the underlying social cohesion, the prescription for each microentrepreneur is to hypothesize that the number of her peers who will opt for a safe investment is a random variable that is uniformly distributed over the unit interval and choose the best action under these circumstances.

As a first step, restrict attention to switching strategies.

Definition 6.3 (Switching strategy)**.** When following a switching strategy around p^*, a borrower invests in a risky project if $p_i \geq p^*$ and invests in a safe project if $p_i < p^*$.

Definition 6.4 (Failure point)**.** Failure point k^* is the threshold value of k below which the group loan just fails. Failure point k^* depends on switching point p^* and *vice versa.*

Suppose all players use switching strategy around p^* and that borrower i's risk premium happens to be exactly p^*. We will derive borrower i's subjective density over the number of group members who make a safe investment choice. That is, borrower i asks herself: "My risk premium is p^*, what is the probability that exactly m out of my n peers will go for a safe project?" Morris and Shin [MS03] show that the answer to this question turns out to be $\frac{1}{n+1}$, irrespective of m. I.e. the probability mass function over the number of players who invest in a safe project is that of a discrete uniform distribution with support $\{0, 1, 2, \ldots, n\}$.[8]

For completeness of the discussion, I reproduce here the proof of discretely and uniformly distributed beliefs from Morris and Shin [MS03], adapting it to the context at hand.

Proposition 6.5. *The probability mass function over the number of peers investing in a safe project is that of a discrete uniform distribution with support* $\{0, 1, 2, \ldots, n\}$ *- conditional on being at the switching point in a switching strategy.*

Proof. Consider figures 6.1 and 6.2. When the common component of the risk premium is θ, the individual risk premia are distributed uniformly over the interval $[\theta - \epsilon, \theta + \epsilon]$. The microentrepreneurs who invest in a safe project are those whose risk premia are below p^*. The probability that a borrower's risk premium is below p^* (and will therefore invest safely) is given by the area under the density to the left of p^*. This area is given by[9]

$$z = \frac{p^* - \theta + \epsilon}{2\epsilon}. \tag{6.1}$$

Thus, the probability that exactly m out of all n peers will go for a safe project is the binomial probability[10]

$$\binom{n}{m} z^m (1 - z)^{n-m},$$

[8]The discrete uniform distribution is also known as the 'equally likely outcomes' distribution and has cumulative distribution function j/n for $j = 1, 2, \ldots, n$.

[9]The probability density function for a continuous uniform distribution on the interval $[a, b]$ is $P(x) = 1/(b - a)$ for $a \leq x \leq b$.

[10]Note the conditions that need to be met for the binomial distribution to hold. (i) Each project choice ('trial') has only two possible outcomes: choice of safe project ('success') or risky project ('failure'); (ii) The probability of choice of a safe project is known and does not vary - here 0.5; (iii) The number of project choices, m, is fixed; (iv) Each project choice is independent.

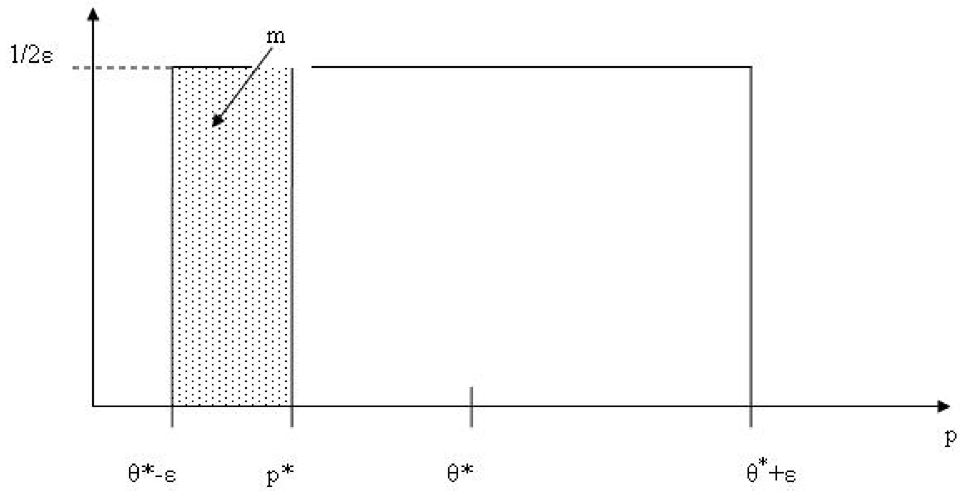

FIGURE 6.1. Deriving the cumulative distribution function $G(k^*|p^*)$. Number of borrower i's peers whose risk premia lie below p^*.

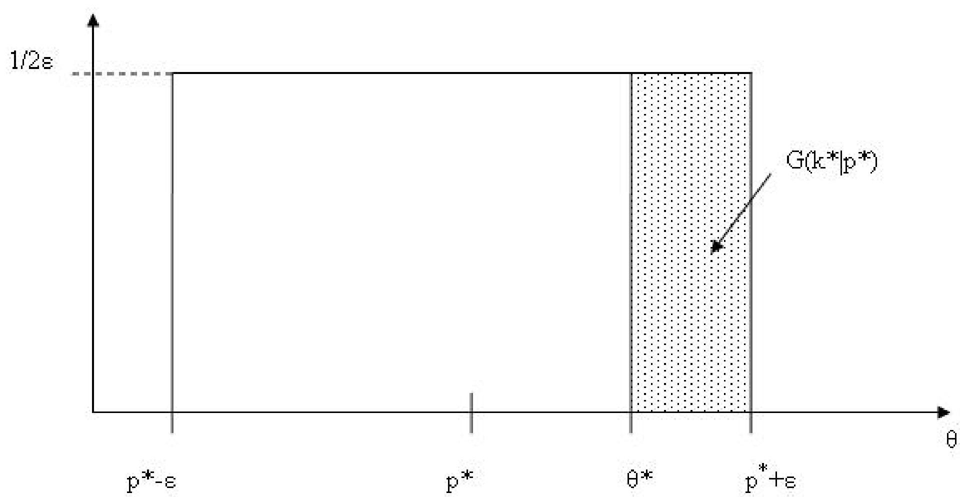

FIGURE 6.2. Deriving the cumulative distribution function $G(k^*|p^*)$. Borrower i's posterior density over θ conditional on her risk premium being p^*.

or, substituting,

$$\binom{n}{m}\frac{(p^*-\theta+\epsilon)^m\,(-p^*+\theta+\epsilon)^{n-m}}{(2\epsilon)^n}. \tag{6.2}$$

We have $k < z$ if and only if $\theta > \theta^*$. Thus, to answer the question above we need to find the probability that $\theta > \theta^*$. We need the posterior density over θ conditional on p^*. This is uniform over

$$[p^* - \epsilon, p^* + \epsilon],$$

since the *ex ante* distribution is uniform and the idiosyncratic element of the risk premium is uniformly distributed around θ. Then

$$\frac{\binom{n}{m}}{2\epsilon} \int_{p^*-\epsilon}^{p^*+\epsilon} \frac{(p^* - \theta + \epsilon)^m (-p^* + \theta + \epsilon)^{n-m}}{(2\epsilon)^n} \, \mathrm{d}\theta. \tag{6.3}$$

We can simplify this expression by taking the change of variables in (6.1). Then $\mathrm{d}\theta = 2\epsilon \, \mathrm{d}z$, while the lower and upper limits of the integral become 0 and 1, respectively. Then (6.3) can be rewritten as

$$\binom{n}{m} \int_0^1 z^m (1-z)^{n-m} \, \mathrm{d}z. \tag{6.4}$$

Appendix 6.4 proves that this integral does not depend on m and is equal to $\frac{1}{n+1}$. $\square$

From the proof above we get that the density over the number of peers who invest in a safe project is uniform, conditional on being at the switching point in a switching strategy. Plus, the expression does not depend on ϵ. Hence, as Morris and Shin [MS02] [MS03] demonstrate: as $\epsilon \to 0$, the uncertainty concerning c dissipates, but the uncertainty over k still remains. That is, even if the underlying fundamentals of the problem were known for sure, strategic uncertainty would still be very severe. The authors also show that this result is not restricted to the uniform-uniform case, but also applies under the assumption of normally-distributed θ and ϵ.

Now consider the reasoning of the borrowers. At switching point p^*, borrower i is indifferent between the risky investment and the safe investment. Recall (5.1); in this case,

$$\mathbb{E}[\lambda_1] = \underline{Z}.$$

The payoff for borrower i in case of group loan failure is independent of how many among her peers opt for a safe project: it is either Y_S, if she goes for a safe project, or $\mathbb{E}[Y_R] - c$, if she goes for a risky project. Therefore, we may consider $\Pr(k < k^*)$. On the other hand, the payoff she gets in case of group loan success varies with the number of other borrowers investing in a safe project. Therefore, we need to consider

$$\sum_{j=k^*}^{n-1} \Pr(k = j).$$

It follows that

$$\begin{aligned}
&\Pr(k=2|p^*)\times\left(Y_S+\overline{Z}\right)+\Pr(k=1|p^*)\times\left(Y_S-\mathbb{E}[\lambda_1]+\overline{Z}\right)\\
&+\Pr(k=0|p^*)\times Y_S\\
=\ &\Pr(k=2|p^*)\times\left(\mathbb{E}[Y_R]-c^*+\underline{Z}\right)+\Pr(k=1|p^*)\times\left(\mathbb{E}[Y_R]-c^*\right)\\
&+\Pr(k=0|p^*)\times\left(\mathbb{E}[Y_R]-c^*\right),
\end{aligned}$$

or

$$\begin{aligned}
&\Pr(k=2|p^*)\times\left(Y_S+\overline{Z}\right)+\Pr(k=1|p^*)\times\left(Y_S-\mathbb{E}[\lambda_1]+\overline{Z}\right)\\
&+\Pr(k<1|p^*)\times Y_S\\
=\ &\Pr(k=2|p^*)\times\left(\mathbb{E}[Y_R]-c^*+\underline{Z}\right)+\Pr(k=1|p^*)\times\left(\mathbb{E}[Y_R]-c^*\right)\\
&+\Pr(k<1|p^*)\times\left(\mathbb{E}[Y_R]-c^*\right),
\end{aligned}$$

or

$$\begin{aligned}
&\Pr(k=2|p^*)\times\left(Y_S+\overline{Z}\right)+\Pr(k=1|c^*)\times\left(Y_S-\mathbb{E}[\lambda_1]+\overline{Z}\right)\\
&+G(k^*|p^*)\times Y_S\\
=\ &\Pr(k=2|p^*)\times\left(\mathbb{E}[Y_R]-c^*+\underline{Z}\right)+\Pr(k=1|p^*)\times\left(\mathbb{E}[Y_R]-c^*\right)\\
&+G(k^*|p^*)\times\left(\mathbb{E}[Y_R]-c^*\right),
\end{aligned}$$

where $G(k^*|p^*)$ denotes the cumulative distribution function over the number of borrowers who go for a safe project, conditional on p^*, evaluated at $k^*=1$.

7. The 3-borrower group example continued

In the 3-borrower example, the probability of 0, 1, or 2 among borrower i's n peers investing in a safe project, conditional on p^*, is invariably $1=(n+1)=1/3$; and $G=k^*/(n+1)=1/3$. We may solve the expression above for c^*:

$$c^*=\mathbb{E}[Y_R]-Y_S-\frac{2\overline{Z}-\underline{Z}-\mathbb{E}[\lambda_1]}{2+k^*} \tag{7.1}$$

With $\underline{Z}=\mathbb{E}[\lambda_1]$, the switching equilibrium is given by

$$p^*=\frac{2z}{2+k^*}. \tag{7.2}$$

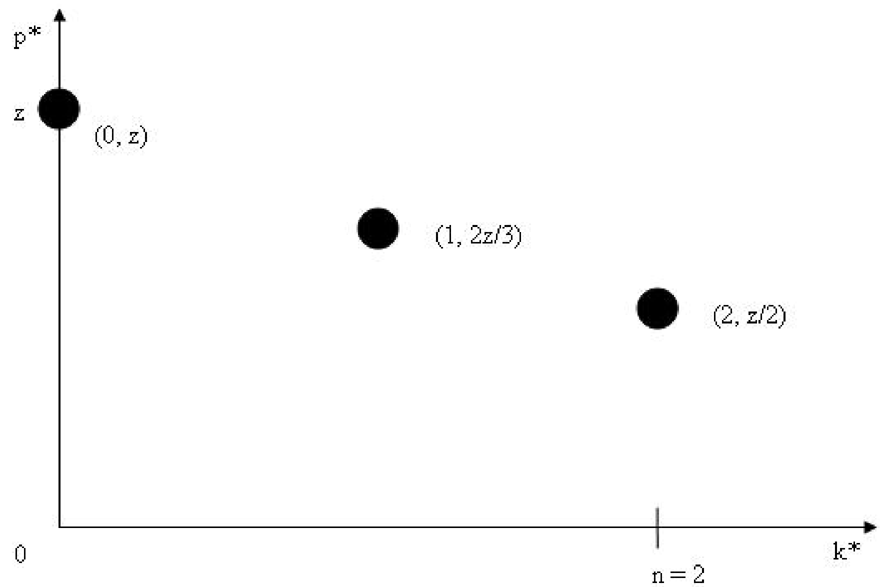

FIGURE 7.1. Switching strategy p^* as a function of critical threshold k^*, for fixed z. The 3-borrower group example.

It results that the threshold point of the switching strategy, p^*, is a decreasing function of the critical threshold k^*. For fixed z, figure 7.1 shows how the switching strategy (as a function of z) changes as the critical threshold changes. For the 3-borrower case, we get values $(0, z)$, $\left(1, \frac{2}{3}z\right)$, and $\left(2, \frac{1}{2}z\right)$ in the (k^*, p^*)-space.

7.1. **Proof of uniqueness of equilibrium.** As borrowers' payoffs satisfy strategic complementarity, uniqueness of the switching equilibrium p^* can be proved by the iterated elimination of dominated strategies from below and above p^*. I here only give a sketch of the proof and refer the reader to Morris and Shin [MS03] for a detailed proof.

Take equation (6.1). Then for p_i lower then the switching strategy p^*, the probability that fewer then k^* peers will go for a safe project decreases. So enough borrowers believe social sanctions are high enough to erode risk premia. This reinforces the idea that borrower i should indeed invest in the safe project, given that her net risk premium is lower than the switching equilibrium.

Say, instead, borrower i has a risk premium higher than the switching value, i.e. $p_i > p^*$, then the probability that fewer then k^* peers will go for a safe project increases. As an increasing number of peers is expected to opt for a risky project, it becomes ever more attractive for borrower i to go for a risky project herself.

8. The $(n+1)$-borrower group example continued

For p^*, borrower i is indifferent between the risky investment and the safe investment, so that

$$\begin{aligned} & \Pr(k=0|p^*)\,\Pi_{i,S}+\ldots+\Pr(k=k^*|p^*)\,\Pi_{i,S}+\ldots+\Pr(k=n|p^*)\,\Pi_{i,S} \\ = \quad & \Pr(k=0|p^*)\,\Pi_{i,R}+\ldots+\Pr(k=k^*|p^*)\,\Pi_{i,R}+\ldots+\Pr(k=n|p^*)\,\Pi_{i,R} \end{aligned}$$

or

$$\begin{aligned} & \sum_{j=0}^{k^*-1}\Pr(k=j|p^*)\,\Pi_{i,S}+G(k^*|p^*)\,\Pi_{i,S} \\ = \quad & \sum_{j=k^*}^{n}\Pr(k=j|p^*)\,\Pi_{i,R}+G(k^*|p^*)\,\Pi_{i,R}. \end{aligned} \tag{8.1}$$

Substituting the payoff expressions in (8.1) gives

$$\begin{aligned} & \sum_{j=k^*}^{n}\Pr(k=j|p^*)\times\left(Y_S+\overline{Z}-\mathbb{E}[\lambda_{n-j}]\right) \\ & +\mathrm{G}(k^*|p^*)\times Y_S \\ = \quad & \sum_{j=k^*}^{n}\Pr(k=j|p^*)\times(\mathbb{E}[Y_R]+\underline{Z}-\mathbb{E}[\lambda_{n-j}]-c^*) \\ & +\mathrm{G}(k^*|p^*)\times(\mathbb{E}[Y_R]-c^*), \end{aligned}$$

with $\mathbb{E}[\lambda_0]=0$.

So we get

$$c^*=\mathbb{E}[Y_R]-Y_S-\frac{(n-k^*+1)\,z-\sum_{j=k^*+1}^{n-1}\mathbb{E}[\lambda_{n-j}]}{n+1}$$

and

$$p^*=\frac{(n-k^*+1)\,z-\sum_{j=k^*+1}^{n-1}\mathbb{E}[\lambda_{n-j}]}{n+1}. \tag{8.2}$$

For fixed z, figure 8.1 shows how the switching strategy (as a function of z) changes as the critical threshold changes. With $n+1$ borrowers, we get values $(0,z-\varphi)$, $\left(1,\frac{n}{n+1}z-\varphi\right)$, $\left(2,\frac{n-1}{n+1}z-\varphi\right)$,... $\left(n,\frac{z}{n+1}-\varphi\right)$ in the (k^*,p^*)-space, with $\varphi=\frac{\sum_{j=k^*+1}^{n-1}\mathbb{E}[\lambda_{n-j}]}{n+1}$.

9. Discussion of results

Let us take the expression (8.2) for the switching equilibrium or, slightly more detailed,

$$\pi_R\,Y_R-Y_S+(1-\pi_R)\,(1+r)\,L-c^*=\frac{(n-k^*+1)\,z-\sum_{j=k^*}^{n-1}\mathbb{E}[\lambda_{n-j}]}{n+1},$$

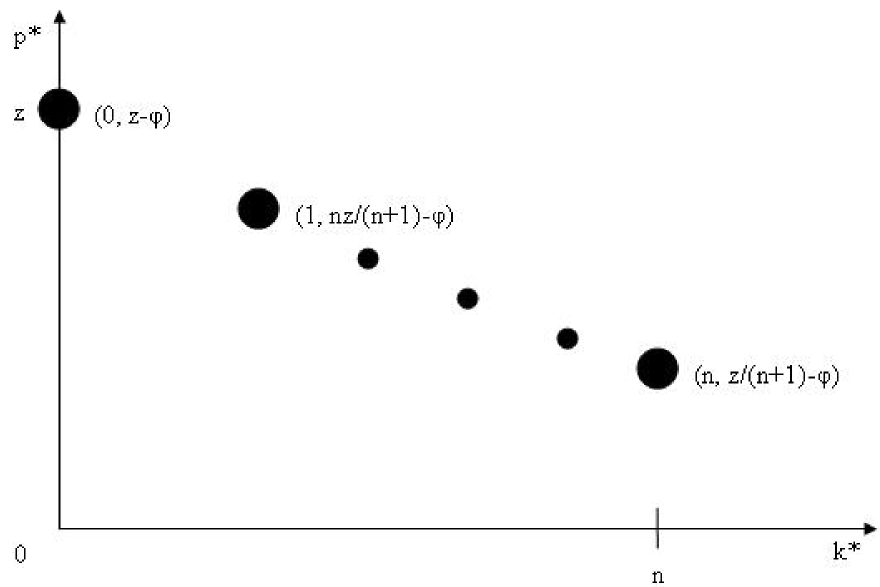

FIGURE 8.1. Switching strategy p^* as a function of critical threshold k^*, for fixed z. The $(n+1)$-borrower group example.

and analyze the effect of its various components.

For larger probability of success of risky projects, π_R, social sanctions need to be very high in order to induce coordination on safe project choices. In figure 6.2 p^* moves further to the right, so that the grey area gets smaller, meaning there is a smaller probability of borrowers investing in a safe project. *Vice versa*, take very risky projects depending, say, on erratic climate conditions, then even small social sanctions are likely to guarantee coordination on the safe project.

The same applies when the potential return from the risky project, Y_R, grows larger. Conversely, when the sure return from the safe project, Y_S, increases, then p^* moves further to the left figure 6.2 and the grey area gets bigger, meaning there is a larger probability of borrowers investing in a safe project.

Keeping other things equal, a higher number of group peers n raises p^* to a value closer to the extra benefit from MFI services z. Indeed,

$$p^* = z - \frac{k^* z + \sum_{j=k^*}^{n-1} \mathbb{E}\left[\lambda_{n-j}\right]}{n+1}.$$

However, a higher n also indirectly lowers p^* by raising the expected compensation amount due to the lender, since more peers $(n - k^*)$ will be undertaking risky projects.

The compensation amount due to the lender itself, λ, too, raises p^*. That is, a higher λ lowers the probability that borrowers will invest in the safe project. This result may also explain why several microfinance lenders, including Muhamed Yunus, have highlighted the disincentive effect of joint liability and have instead promoted flexible joint liability, where the compensation amount is optimally determined in each single case.

The nominal loan amount and the interest charged on the loan matter as well. Indeed, as L and / or r increase, so does p^*. So it seems that lower loan amounts induce cooperative behavior; this may explain why for relatively high amounts, microentrepreneurs apply for (and MFIs hand out) individual rather than group loans. On the one hand, a microentrepreneur avoids the high risk of peers not cooperating; on the other hand, the MFI avoids the risk of lending to a group with insufficient creditworthiness.

10. Conclusions

The main contribution of my paper has been to provide a new application of the theory of global games, expanding it to the study of microfinance group lending. With the exception of a single paper by Bond and Rai [BR09] in the development economics literature, mine is indeed the first analysis of solidarity group lending in terms of a global game. Through a rich payoff structure, I have analyzed the extent to which social cohesion in a community of microentrepreneurs motivates cooperation towards success of a group loan.

Compared to previous analyses of the cooperative behavior of microfinance borrowers, the global game framework highlights the role of strategic uncertainty. Even in the absence of uncertainty regarding the fundamentals of the business environment, uncertainty regarding the actions taken by one's peers may still be severe. The switching equilibrium, at which a group member decides to go for a safe project - thereby contributing towards the success of the group loan - is a function of the number of her peers she believes will also go for a safe project.

The risk premium on alternative risky projects and the threat of social sanctions play an important role. I have illustrated the conditions under which coordination on the socially optimal outcome is easier to achieve: when risky projects have a low probability of success and/or yield low expected returns; when safe projects yield high sure returns; when loan amounts and interest rates are relatively low; and when the MFI's business support services have a high (non-monetary) value. This last point highlights the importance of non-monetary benefits accompanying microfinance loans.

The results I have obtained in this paper can be of special interest to microfinance lenders. The main issue is the extent to which the debt capacity of *groups* of borrowers, rather than that of individual borrowers, depends on the strength of social cohesion. I have also highlighted how this model may be of use to determine the loan amount up to which group loans work better than individual loans, and *vice versa*. But further investigation is required to clarify the role played here by social cohesion.

A final question is whether fear of social ostracism or benefiting from MFI services matters more in inducing cooperative behavior among group loan members. In terms of the model at hand, this question concerns the relative importance of c^* compared to Z. Here again, further research is needed to answer this question.

References

[AM05] B. Armendariz and J. Morduch. *The Economics of Microfinance*. MIT Press, 2005.

[AMS08] A. Ambrus, M. Mobius, and A. Szeidl. Consumption risk-sharing in social networks. Working paper, Harvard University, 2008.

[BO09] B. Bhole and S. Ogden. Group lending and individual lending with strategic default. *Journal of Development Economics*, 00:00–00, 2009.

[BR09] P. Bond and A. Rai. Borrower runs. *Journal of Development Economics*, 88:185–191, 2009.

[CCW07] A. Cassar, L. Crowley, and B. Wydick. The effect of social capital on group loan repayment: Evidence from field experiments. *Economic Journal*, 117:F85–F106, 2007.

[CvD93] H. Carlsson and E. van Damme. Equilibrium selection in stag hunt games. In K. Binmore, A. Kirman, and A. Tani, editors, *Frontiers of Game Theory*. MIT Press, 1993.

[CW09] A. Cassar and B. Wydick. Does social capital matter? Evidence from a five-country group lending experiment. Working paper, University of San Francisco, 2009.

[DJ01] J. Dayton-Johnson. *Social Cohesion and Economic Prosperity*. James Lorimer & Company, 2001.

[GJKM06] X. Giné, P. Jakiela, D. Karlan, and J. Morduch. Microfinance games. Working paper, World Bank, 2006.

[GS03] R. Gomez and E. Santor. Do peer group members outperform individual borrowers? A test of peer group lending using Canadian micro-credit data. Working Paper, Bank of Canada, 2003.

[Kar07] D. Karlan. Social connections and group banking. *Economic Journal*, 117:F52–F84, 2007.

[KMRS09] D. Karlan, M. Mobius, T. Rosenblat, and A. Szeidl. Trust and social collateral. *Quarterly Journal of Economics*, 124:1307–1361, 2009.

[MS02] S. Morris and H. Shin. Measuring strategic uncertainty. Working Paper, 2002.

[MS03] S. Morris and H. Shin. Global games: Theory and applications. In M. Dewatripont, L. Hansen, and S. Turnovsky, editors, *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society*. Cambridge University Press, 2003.

[Osb03] L. Osberg. *The Economic Implications of Social Cohesion*. University of Toronto Press, 2003.

[Sti90] J. Stiglitz. Peer monitoring and credit markets. *World Bank Economic Review*, 4:351–366, 1990.

[WHK09] B. Wydick, H. Hayes, and S. Kempf. Social networks, neighborhood effects, and credit access: Evidence from rural Guatemala. *World Development*, 2009.

[Wyd99] B. Wydick. Can social cohesion be harnessed to repair market failures? Evidence from group lending in Guatemala. *Economic Journal*, 109:463–475, 1999.

Appendix A. Payoffs for the 3-player, 2-action game

For a 3-player (i, j, l), 2-action (S, R) game, the payoffs are as follows.

$$\begin{aligned}(\Pi_{i,S};\Pi_{j,S};\Pi_{l,S}) &= \big((Y_S-(1+r)L)+\overline{Z};\\ &\quad (Y_S-(1+r)L)+\overline{Z};\\ &\quad (Y_S-(1+r)L)+\overline{Z}\big)\,.\end{aligned}$$

$$\begin{aligned}(\Pi_{i,S};\Pi_{j,S};\Pi_{l,R}) &= \Bigg(\pi_R\,(Y_S-(1+r)L)+(1-\pi_R)\left(Y_S-\frac{3}{2}(1+r)L\right)+\overline{Z};\\ &\quad \pi_R\,(Y_S-(1+r)L)+(1-\pi_R)\left(Y_S-\frac{3}{2}(1+r)L\right)+\overline{Z};\\ &\quad \pi_R\,(Y_R-(1+r)L)-c+\underline{Z})\,.\end{aligned}$$

$$\begin{aligned}(\Pi_{i,S};\Pi_{j,R};\Pi_{l,S}) &= \Bigg(\pi_R\,(Y_S-(1+r)L)+(1-\pi_R)\left(Y_S-\frac{3}{2}(1+r)L\right)+\overline{Z}\\ &\quad \pi_R\,(Y_R-(1+r)L)-c+\underline{Z};\\ &\quad \pi_R\,(Y_S-(1+r)L)+(1-\pi_R)\left(Y_S-\frac{3}{2}(1+r)L\right)+\overline{Z}\Bigg)\,.\end{aligned}$$

$$\begin{aligned}(\Pi_{i,S};\Pi_{j,R};\Pi_{l,R}) &= ((Y_S-(1+r)L)\,;\\ &\quad \pi_R\,(Y_R-(1+r)L)-c;\\ &\quad \pi_R\,(Y_R-(1+r)L)-c)\,.\end{aligned}$$

$$\begin{aligned}(\Pi_{i,R};\Pi_{j,S};\Pi_{l,S}) &= (\pi_R\,(Y_R-(1+r)L)-c+\underline{Z};\\ &\quad \pi_R\,(Y_S-(1+r)L)+(1-\pi_R)\left(Y_S-\frac{3}{2}(1+r)L\right)+\overline{Z};\\ &\quad \pi_R\,(Y_S-(1+r)L)+(1-\pi_R)\left(Y_S-\frac{3}{2}(1+r)L\right)+\overline{Z}\Bigg)\,.\end{aligned}$$

$$\begin{aligned}(\Pi_{i,R};\Pi_{j,S};\Pi_{l,R}) &= (\pi_R\,(Y_R-(1+r)L)-c;\\ &\quad (Y_S-(1+r)L)\,;\\ &\quad \pi_R\,(Y_R-(1+r)L)-c)\,.\end{aligned}$$

$$\begin{aligned}(\Pi_{i,R};\Pi_{j,R};\Pi_{l,S}) &= (\pi_R\,(Y_R-(1+r)L)-c;\\ &\quad \pi_R\,(Y_R-(1+r)L)-c;\\ &\quad (Y_S-(1+r)L))\,.\end{aligned}$$

$$\begin{aligned}(\Pi_{i,R};\Pi_{j,R};\Pi_{l,R}) &= (\pi_R\,(Y_R-(1+r)L)-c;\\ &\quad \pi_R\,(Y_R-(1+r)L)-c;\\ &\quad \pi_R\,(Y_R-(1+r)L)-c)\,.\end{aligned}$$

Appendix B. Proof of (6.4)

Proof as in Morris and Shin [MS03] that the integral (6.4) does not depend on m.

Proof. Begin by noting that

$$\begin{aligned} z^m(1-z)^{n-m} &= z^m \sum_{i=0}^{n-m} \binom{n-m}{i} (-1)^i z^i \\ &= \sum_{i=0}^{n-m} \binom{n-m}{i} (-1)^i z^{m+i}. \end{aligned}$$

Thus

$$\int z^m(1-z)^{n-m}\,\mathrm{d}z = \sum_{i=0}^{n-m} \binom{n-m}{i} (-1)^i \frac{z^{m+i+1}}{m+i+1},$$

so that

$$\int_0^1 z^m(1-z)^{n-m}\,\mathrm{d}z = \sum_{i=0}^{n-m} \frac{\binom{n-m}{i} (-1)^i}{m+i+1}.$$

However,

$$\begin{aligned} \sum_{i=0}^{n-m} \frac{\binom{n-m}{i} (-1)^i}{m+i+1} &= \frac{m!\,(n-m)!}{(n+1)!} \\ &= \frac{1}{(n+1)\binom{n}{m}}. \end{aligned}$$

Thus

$$\binom{n}{m} \int_0^1 z^m(1-z)^{n-m}\,\mathrm{d}z = \frac{1}{n+1}.$$

□

Zürcher Kantonalbank
E-mail address: miret.padovani@zkb.ch

VALUE CHAIN FINANCING AND LOCAL INTERACTION GAMES

MIRET PADOVANI

ABSTRACT. Value chain financing is of great importance to the agricultural sector of most developing countries. Microfinance institutions may contribute to the strength of chain links by handing out loans. I view an agricultural value chain as a network, where links between nodes denote social and business relationships between chain actors. By studying internode relationships in terms of local interaction games, I address the issue of whether and how network structure and social cohesion may spread coordination on safe investment choices from a local level to the entire value chain.

1. INTRODUCTION

Ever since the take-off of microfinance in the seventies, microfinance institutions (MFIs) have been involved in several innovative financing schemes. Their goal has been to improve the livelihoods of poor communities, by endorsing the entrepreneurial skills of people who would otherwise not have access to banking facilities. In this paper I take a game-theoretic look at agricultural value chain financing, a field in which MFIs are becoming increasingly involved.

A value chain is a supply chain consisting of the input suppliers, producers, processors, and buyers that bring a product from its conception to its end use. Value chain financing is of great importance to the economy of most developing countries.[1] A recent conference by the Food and Agriculture Organization (FAO) on "Financing agricultural value chains" has highlighted the risks faced by independent farmers:[2]

> "[I]ndependent farmers, faced with dramatic price risks, will eventually become broken links in fragmented chains, unable to survive competition. [...] [T]oday's markets require integrated systems of differentiated production in which farmers, processors and marketers work interdependently. These producers can become and remain competitive if they have modern, well-organized chains and dynamic, flexible financial services."

The role of MFIs in value chain financing has been steadily increasing, as Piana [Pia08] ascertains in a recent study:

> "MFIs in different locations can turn out to finance broad and overlapping groups of businesses, who could become suppliers and

Date: First version: January 2010. This version: February 18, 2010.

Key words and phrases. Value chain financing; microfinance; local interaction games; networks.

I thank Hyun Song Shin and Paolo Vanini for several discussions on this topic and helpful comments on earlier drafts of this paper.

[1]See Fries and Akin [FA04] on the significance of value chains in rural finance.

[2]See Quiros [Qui07] for the conference papers.

> clients to each other. It's frequently the case that, in different times and across different MFIs, their customers comprehend agricultural growers, small processors, craftsmen and manufacturers, street vendors, small shops, and even transport and logistic operators as well as other specialised input and service providers."

MFIs hand out a variety of loan types, which can be categorized into individual loans, group loans, and village loans. Individual loans tend to be awarded to larger businesses and for larger sums of money. MFIs also assist their clients in setting up their new businesses.

As microentrepreneurs typically lack physical collateral which they can pledge against the loans they receive, MFIs need to make use of collateral substitutes. An MFI can in most settings exploit the social sanction opportunities that exist between borrowers. Social collateral generated with group formation and imposition of joint responsibility at the group level may be deemed as an effective substitute for conventional creditworthiness. As such, social collateral offers a guarantee for loan repayment and minimizes potential loan defaults.

Social sanctions may take several forms. Defecting borrowers may be excluded from privileged access to input supplies, from further trade credit, from social and religious events, or from day-to-day courtesies. The threat of such sanctions is more credible the smaller the village community.[3] Indeed, the strength of social sanctions mirrors the strength of social cohesion, i.e. the network of social relationships binding people together in communities and neighborhoods.

It is hard to give a precise definition of social collateral, given its multiple facets. Nevertheless, it is worthy to note at this stage the distinction made in the sociology literature between social capital and social cohesion, both of which constitute a type of social collateral:

> "Social capital is a characteristic of the individual, defined as the current sacrifice in time, effort, or money made by that individual in the hope of promoting productive cooperation or coordination with others (Osberg [Osb03])."

On the other hand,

> "Social cohesion is a characteristic of the society or collective that depends on the history of the accumulation of social capital in the group and which, in turn, affects the incentives for current social capital investments (Osberg [Osb03])."

Both aspects of social collateral are crucial for the success of microfinance: social capital determines the creditworthiness of individual borrowers, social cohesion that of an entire group of borrowers. My interest in this paper focusses on the latter.[4]

The literature on the role of social collateral in microfinance lending has been to date confined to solidarity group lending, where 5 to 6 microentrepreneurs share joint liability. With a very few exceptions,[5] it also abstains from any network structure considerations. Yet the findings are also interesting for the study of value chain financing in developing countries. Empirical studies give, however, a mixed

[3]See discussion in Armendariz and Morduch [AM05].

[4]Osberg [Osb03] analyzes the economic implications of social cohesion. This field of research is concerned with the question of whether differences in trust, social capital, social cohesion can explain differences in the economic performance of world regions. See also Dayton-Johnson [DJ01].

[5]See Ambrus *et al.* [AMS08], on which I say more below.

picture concerning the extent to which social cohesion affects the performance of group lending. Differences in results sometimes seem to depend on the proxy used. Wydick [Wyd99] shows that groups of strangers can do as well as groups of friends - if not even better. Social ties *per se* have little impact on the performance of borrowing groups. What really matter are peer monitoring and group pressure. Peer monitoring denotes the ability of borrowers to monitor the investment behavior of another during the course of the loan, making sure that each group member undertakes only safe investment projects with borrowed capital; whereas, group pressure denotes the ability to threaten exclusion of non-paying members from continued access to credit. Since friends may be softer and more forgiving on each other, it is not necessarily true that stronger social ties lead to higher repayment rates.

On the other hand, subsequent field experiments conducted in several developing countries by the same author,[6] as well as research by Gomez and Santor [GS03] demonstrate that the strength and credibility of the threat of social sanctions against defaulting members is the key determinant in group loan performance.

Ambrus *et al.* [AMS08] bring network structure into the picture and study the effectiveness of network-based insurance among microentrepreneurs. The authors ask when, and to what extent, do networks allow local shocks to be shared globally.

While the inspiration for my work is the literature on the extent to which social cohesion and the threat of social sanctions may explain high repayment rates on microfinance loans,[7] the modeling framework is related to the literature on local interaction games on networks. Local interaction games consist of a local interaction system describing how players interact and the payoffs they derive from those interactions. I apply to a microfinance context the game framework formalized by Morris [Mor00], who addresses the question of whether there exists a finite group of players, such that if that group starts out playing some action, best response dynamics will ensure that that action is eventually played out everywhere. The question I ultimately aim to answer is whether and how network structure and social cohesion may spread coordination on the socially desirable outcome from a local level to the entire value chain.[8] To the best of my knowledge, there has never been an analysis of microfinance loans for value chain financing in terms of local interaction games on networks.

The remainder of the paper is organized as follows. The next section describes the microfinance local interaction game taking place between chain actors. Sections 3 to 6 analyze several network structures and their corresponding contagion thresholds. Section 8 discusses group properties leading to a higher probability of contagion. Section 9 concludes and suggests further research.

2. Microfinance local interaction games

Direct financial agreements between actors on a value chain are very common in both developing and developed economies. This is typically known as trade credit. In developing countries, the financial flows often take the form of in-kind

[6] See Cassar *et al.* [CCW07], Cassar and Wydick [CW09], and Wydick *et al.* [WHK09].

[7] See Karlan [Kar07], Karlan *et al.* [KMRS09], and Ambrus *et al.* [AMS08].

[8] There is a burgeoning literature on different applications of network theory to financial markets. For a survey on networks in finance see Allen and Babus [AB09]. Experimental literature on coordination in financial networks includes Gale and Kariv [GK08].

transfers: e.g. a lender may advance fertilizer for payment at a later date, then receive payment in the form of produce. So-called 'indirect' value chain financing is instead composed of financing from banks or MFIs. Such funding is the focus of my paper. Loans received from financial institutions offer microentrepreneurs several benefits over informal or peer-to-peer loans: longer term investment capital rather than just short term working capital; larger nominal amounts; more transparent loan pricing; more secure markets, etc.

I analyze a value chain in terms of its network structure. I apply to a microfinance context the local interaction game framework formalized by Morris [Mor00]. A local interaction game consists of a local interaction system describing how players interact and their payoffs from those interactions.

In the context of this paper, a local interaction system consists of a countably infinite set $\mathcal{H}$ of microentrepreneurs (the 'players') involved in a value chain and a binary relation $\sim$ on this set. The relation $\sim$ denotes a business and possibly also social link between two chain actors. In network jargon: if $i \sim j$, node i is said to be a neighbor of node j. The value of each link is denoted by c_{ij} and corresponds to the amount of social collateral a microentrepreneur can 'pledge' against a loan she takes from an MFI.

So a local interaction system is a pair $(\mathcal{H}, \sim)$, with $\sim$ satisfying the following four properties:[9]

1. Irreflexivity: No player is his own neighbor.
2. Symmetry: If i is a neighbor of j, then j is a neighbor of i.
3. Bounded neighbors: Each player has at most a finite number of neighbors.
4. Connectedness: There is some path connecting any pair of players.

I abide by the notation by Morris [Mor00] and denote by Γ_i the set of neighbors of microentrepreneur i, i.e. $\Gamma_i \equiv \{x' : x' \sim x\}$. The group X, a subset of $\mathcal{H}$, is the group of peers with whom i enters into business transactions.

Each microentrepreneur is given a loan with face value L and can undertake one of two projects: either a safe project, yielding Y_S for sure (i.e. with probability of success $\pi_S = 1$), or a risky project, yielding $Y_R > Y_S$ with probability of success $\pi_R < 1$ and 0 otherwise. A borrower has to repay her loan only if her project is successful; she may not use wealth from other investments to repay the current investment's loan. Undertaking a safe project is the socially desirable outcome, since it contributes to successful continuation of the value chain. The risk premium represents, therefore, the tension between the socially desirable outcome and the outcome resulting from the self-interested action of chain actors.

Note that we may also think of households, rather than individual borrowers. E.g. a household owning a coffee bean farm. Investment decisions are taken by the household as a whole and there is no need to model coordination within the household itself. But for the remainder of the paper, I will simply refer to nodes as 'borrowers'.

The success of borrowers' projects are independent. Borrowers do not communicate their investment choices and *ex ante* observation of investment choices is not possible. This assumption begs the question of whether ex ante observation is truly never possible in microfinance investments, or only sometimes. Though the nature of the business may very well be observable (e.g. producing scarves),

[9]For the technical details of these properties, see Morris [Mor00].

the instruments used for production, for example, may not be. So using defective machinery can be interpreted as undertaking a risky investment.

Also note that the assumption of projects' outcomes being independent does not mean that the businesses themselves are independent - they do, indeed, belong to the same value chain. The independence assumption refers to the fact that the risk source (which is not a priori observable) does not depend on the characteristics of the value chain and of its linkages.

After *ex post* observation of a risky choice taken by chain actor i, her neighbor j can impose on her a social sanction, which is represented by the loss of link value c_{ij}. Let us denote by $\Pi_{a,a'}$ the payoff of a microentrepreneur from a particular interaction if she invests in project a and her neighbor invests in project a'. This payoff function leads us to the following symmetric payoff matrix:

i; j	S	R
S	$(\Pi_{S,S}; \Pi_{S,S})$	$(\Pi_{S,R}; \Pi_{R,S})$
R	$(\Pi_{R,S}; \Pi_{S,R})$	$(\Pi_{R,R}; \Pi_{R,R})$

The payoffs in the matrix above are given by

$$(\Pi_{S,S}; \Pi_{S,S}) = (Y_S - (1+r)L + \Delta_c;\ Y_S - (1+r)L + \Delta_c)\,,$$
$$(\Pi_{R,S}; \Pi_{S,R}) = (\pi_R\ (Y_R - (1+r)L) - c_{ij};\ Y_S - (1+r)L - c_{ij})\,,$$
$$(\Pi_{S,R}; \Pi_{R,S}) = (Y_S - (1+r)L - c_{ij};\ \pi_R\ (Y_R - (1+r)L) - c_{ij})\,,$$

and

$$(\Pi_{R,R}; \Pi_{R,R}) = (\pi_R\ (Y_R - (1+r)L) - c_{ij};\ \pi_R\ (Y_R - (1+r)L) - c_{ij})\,.$$

The term Δ_c denotes the additional link value added to a link (i, j) when interaction between microentrepreneurs i and j is successful. I.e. Δ_c is added to the social capital accumulated until then, c_{ij}, and the new value of the link becomes $\Delta_c + c_{ij}$.

It is assumed that this game has two strict Nash equilibria, namely "Both invest in safe projects" and "Both invest in risky projects", so that

$$\Pi_{S,S} > \Pi_{R,S};\ \Pi_{R,R} > \Pi_{S,R}.$$

Hence, link value c_{ij} satisfies

$$Y_S - (1+r)L + \Delta_c \geq \pi_{i,R}\ (Y_R - (1+r)L) - c_{ij} \geq Y_S - (1+r)L.$$

The risk premium net of social sanctions lies somewhere within the interval $[0, \Delta_c]$. Since

$$Y_S - (1+r)L + \Delta_c \geq \pi_{i,R}\ (Y_R - (1+r)L) - c_{ij},$$

the equilibria are Pareto-rankable and it is in both microentrepreneurs' interest to coordinate on undertaking safe investments. A coordination failure arises when the Pareto-superior equilibrium fails to be selected.

Remark 2.1. In the remainder of the paper, I assume all links have the same value $c_{ij} = c$ and can be increased by the same amount Δ_c. I also simplify the notation as follows.

- $\mathbb{E}[Y_R] = \pi_R (Y_R - (1+r) L)$, the expected return on the risky project - after paying back the loan principal plus interest;
- $Y_S = Y_S - (1+r) L$, the sure return on the safe project - after paying back the loan principal plus interest;
- $P = \mathbb{E}[Y_R] - Y_S$, the gross risk premium.

When the risk premium net of social sanctions lies within the dominance thresholds derived above, 0 and Δ_c, investment in a safe project is a best response for chain actor i if she assigns at least probability

$$q = \frac{D - C}{(A - C) + (D - B)}$$

to her neighbor j investing in a safe project, too. Substituting, q is given by

$$q = \frac{\mathbb{E}[Y_R] - c - Y_S}{\Delta_c - c}.$$

Now a local interaction game is a 3-tuple $(\mathcal{H}, \sim, q)$.

Following Morris [Mor00], I describe best responses as follows. A configuration is a function s mapping the set of chain actors to the set of possible investments choices, i.e. $s : \mathcal{H} \to \{S, R\}$. Given configuration s, chain actor i's best response is to invest in the project that maximizes the sum of her payoffs from her interactions with each of her neighbors. Therefore, investing in a safe project is a best response to configuration s for microentrepreneur i if

$$\sum_{j \in \Gamma(i)} \Pi_{S,s(j)} \geq \sum_{j \in \Gamma(i)} \Pi_{R,s(j)}.$$

Then investment in a safe project is a best response for borrower i if she believes at least proportion q of her neighbors on the network will invest in a safe project. Investment in a risky project is a best response if at least proportion $(1 - q)$ of her neighbors invest in a risky project.

2.1. **Contagion on networks.** Network studies are mostly concerned with either one of two aspects: the process by which networks form ('connection') or the way networks operate to influence nodes' behavior ('contagion'). The seminal work on threshold models for collective behavior and contagion in social networks is that by Granovetter [Gra78].[10] The concept of 'contagion threshold' is widespread in several disciplines, ranging from epidemiology to viral marketing. As an example, the 'epidemic threshold' studied in epidemiology depicts the critical rate of infection for epidemic transition. But the notion of network contagion has a variety of interesting applications in business and finance, too, such as when studying the diffusion of financial innovations.

In this paper, I consider contagion of safe investment choices made by value chain actors. Take, as an example, an Ethiopian household managing a coffee bean washing station.[11] To set up its business, the household requires a washing machine, an artificial dryer, plus an automobile for transporting the washed beans to export firms. One alternative for the household is to take an individual loan for the entire funding amount it needs; another alternative is to enter a group loan, where it would share jointly liability with other microentrepreneurs. The loans I consider in this paper are individual liability loans.

[10] Gladwell [Gla00] provides an interesting introduction on social epidemics.
[11] On coffee growing in Ethiopia, see the FAO's document at link.

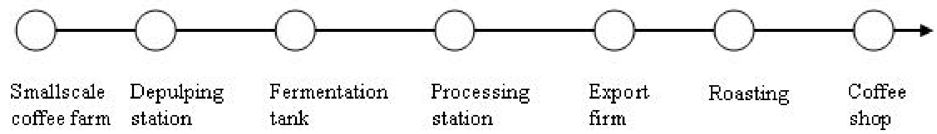

FIGURE 3.1. Interaction on a line network. Example of a coffee value chain.

Note that, even in absence of joint liability, it is important for chain actors to successfully coordinate on investing the borrowed funds safely. Only when initial loans are paid back, does an MFI offer business assistance to its clients and the opportunity of further loans at more favorable terms. These benefits positively feed back to the entire value chain. If the good outcome spreads throughout the network, then this contributes to a successful value chain - from coffee production to coffee marketing.

Employing the terminology of the local interaction games framework formalized by Morris [Mor00], the contagion threshold ξ is the largest q such that, if some finite group of players starts out playing some action (say, choosing safe investment projects), best response dynamics will ensure that that action is eventually played everywhere.

Definition 2.2 (Contagion threshold)**.** The contagion threshold ξ of local interaction system $(\mathcal{H}, \sim)$ is the largest q such that choice of safe investment projects spreads by best response from some finite group to the entire value chain.

In the following sections, I analyze microfinance games on specific network structures. In doing so, I choose network structures which suitably describe value chains and give the contagion threshold in terms of social cohesion within a microfinance context.

3. Interaction on a line

I start by considering interaction on a simple line network. As figure 3.1 illustrates through the example of the labor intensive coffee value chain, microentrepreneurs are arranged along a line and each one of them enters into business agreements with the individual to her left and the one to her right. This network structure can be useful to describe a very simple chain, where every activity on the chain is carried out by a single microentrepreneur or household and every chain actor has a single supplier and a single buyer.

When microentrepreneurs interact on a line network, there are - from microentrepreneur i's perspective - three possible configurations: neither neighbor invests in a safe project; only one neighbor invests in a safe project; both neighbors invest in a safe project. The following conditions need to hold under each possible configuration so that it is a best response for microentrepreneur i to invest in a safe project:

- Given the configuration under which neither neighbor invests in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S - 2c > \mathbb{E}[Y_R] - 2c.$$
Rearranging,
$$0 > P,$$
but we know that a negative risk premium is implausible. Hence, this condition cannot hold.
- Given the configuration under which only one neighbor invests in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S + \Delta_c - c > \mathbb{E}[Y_R] - 2c,$$
or
$$c + \Delta_c > P.$$
- Given the configuration under which both neighbors invest in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S + 2\Delta_c > \mathbb{E}[Y_R] - 2c,$$
or
$$2(c + \Delta_c) > P.$$

These conditions can be generalized into the following proposition, which not only applies to the line network, but also - as I will show below - to all other network structures considered in this paper.

Proposition 3.1. *Given the configuration under which x neighbors invest in a safe project, it is a best response for microentrepreneur i to invest in a safe project if the risk premium is lower than $x(c + \Delta_c)$.*

Corollary 3.2. *Given the configuration under which neither neighbor invests in a safe project, it is never a best response for microentrepreneur i to invest in a safe project, since that would require the implausible condition of a negative risk premium.*

Corollary 3.2 highlights the irrelevance of the threat of social sanctions in inducing cooperation on the socially best outcome when all neighbors act egoistically.

3.1. **Contagion on a line.** Morris shows that the contagion threshold for interaction on a line is $\xi = \frac{1}{2}$.

If $q < \frac{1}{2}$, then investment in a safe project is a best response whenever at least one neighbor invests in a safe project, too. Thus, if two neighboring chain actors i and $i+1$ invest in a safe project in the initial period, actors $i-1$, i, $i+1$, and $i+2$ must all go for a safe investment in the next period, actors $i-2$, $i-1$, i, $i+1$, $i+2$, and $i+3$ must all go for a safe investment in the following period, and so on.

But if $q > \frac{1}{2}$, then investment in a safe project is a best response only when both neighbors invest in a safe project.

In this context, $q < \frac{1}{2}$ translates into the following condition on the risk premium:
$$\frac{\mathbb{E}[Y_R] - c - Y_S}{\Delta_c - c} < \frac{1}{2},$$

or

$$\frac{1}{2}\left(\Delta_c + c\right) > P;$$

which leads us to proposition 3.3.

Proposition 3.3. *Contagion of safe investment decisions occurs throughout the line-shaped value chain when the risk premium is lower than* $\frac{1}{2}\left(\Delta_c + c\right)$.

4. Interaction on a binary tree

Define a walk as a sequence of links connecting a sequence of nodes. A cycle is a walk that starts and ends at the same node, with all nodes appearing once except the starting node, which also appears as the ending node. A tree is a connected network that has no cycles. As figure 4.1 illustrates, each node is linked to n nodes to its right. For $n = 2$, we get a binary tree, as in figure 4.2.

The tree structure can, too, represent a value chain, where every chain actor has one supplier but several buyers. The chain is then composed of interrelated sequential *and* parallel functions involved in the production, manufacturing, and marketing of goods. Having more links on a binary tree than on a line means that each chain actor has more social collateral to provide and can, therefore, take on more credit to finance her business.

When microentrepreneurs interact on a binary tree, there are - from microentrepreneur i's perspective - four possible configurations: neither neighbor invests in a safe project; one neighbor invests in a safe project; two neighbors invest in a safe project; all three neighbors invest in a safe project. The following conditions need to hold under each possible configuration so that it is a best response for microentrepreneur i to invest in a safe project:

- Given the configuration under which neither neighbor invests in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S - 3c > \mathbb{E}\left[Y_R\right] - 3c,$$
which - as seen above - cannot hold, since it would imply a negative risk premium.
- Given the configuration under which one neighbor invests in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S + \Delta_c - 2c > \mathbb{E}\left[Y_R\right] - 3c,$$
or
$$c + \Delta_c > P.$$
- Given the configuration under which two neighbors invest in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S + 2\Delta_c - c > \mathbb{E}\left[Y_R\right] - 3c,$$
or
$$2\left(c + \Delta_c\right) > P.$$
- Given the configuration under which all three neighbors invest in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S + 3\Delta_c > \mathbb{E}\left[Y_R\right] - 3c,$$

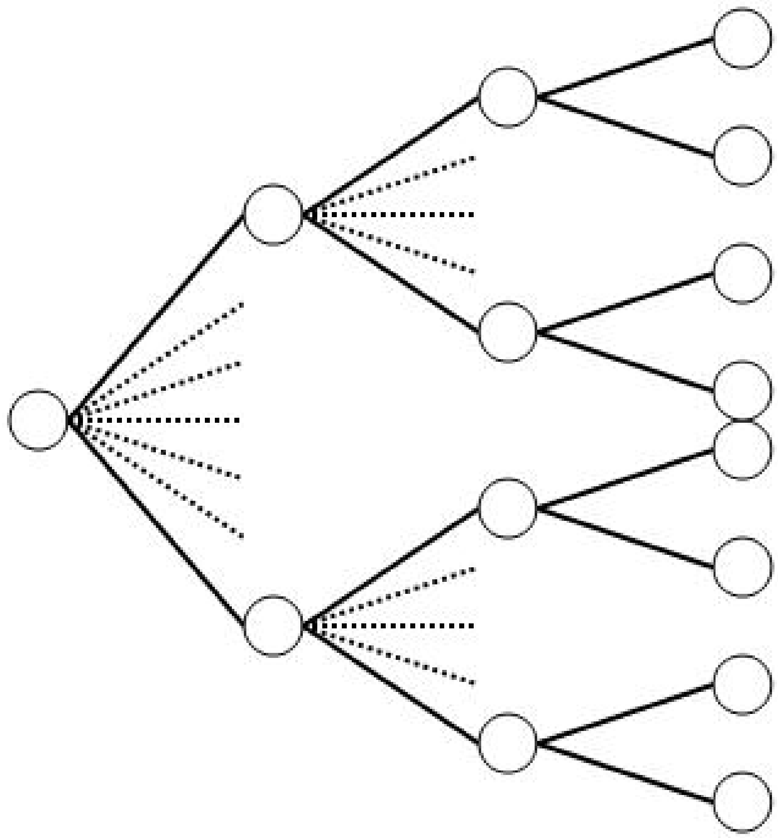

FIGURE 4.1. Interaction on a multiple-branch tree.

or

$$3\,(c + \Delta_c) > P.$$

4.1. **Contagion on a binary tree.** Morris shows that the contagion threshold for interaction on a binary tree is $\xi = \frac{1}{3}$. So if $q < \frac{1}{3}$, then investment in a safe project is a best response whenever at least one neighbor invests in a safe project, too.

This requires the risk premium to fulfill the following condition:

$$\frac{\mathbb{E}\,[Y_R] - c - Y_S}{\Delta_c - c} < \frac{1}{3},$$

or

$$\frac{1}{3}\Delta_c + \frac{2}{3}c > P;$$

which leads us to proposition 4.1.

Proposition 4.1. *Contagion of safe investment decisions occurs throughout the binary tree value chain when the risk premium is lower than* $\frac{1}{3}\Delta_c + \frac{2}{3}c$.

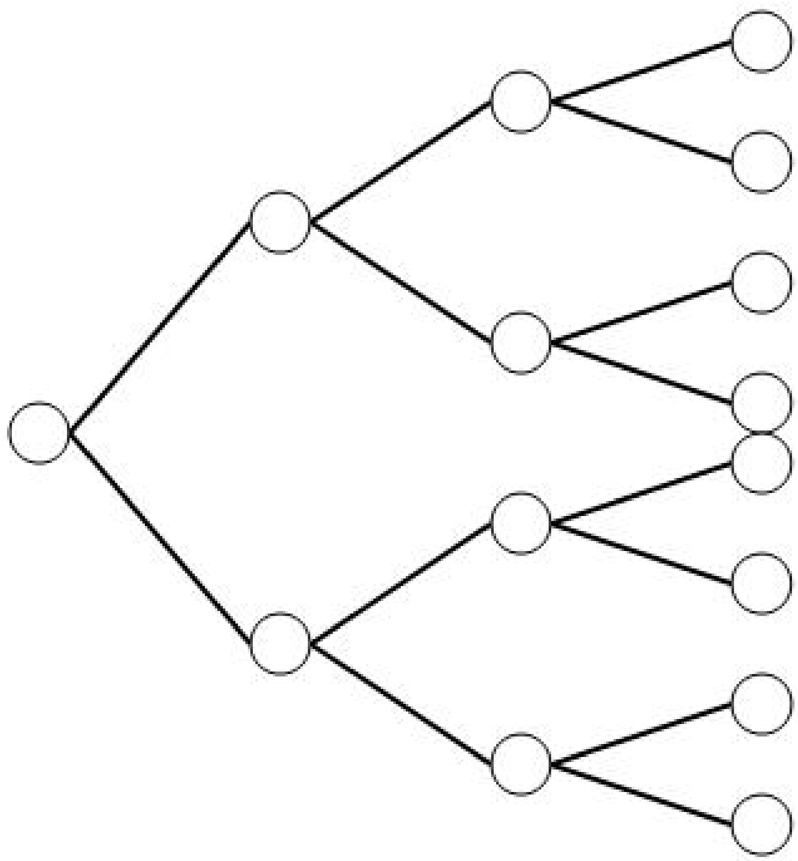

FIGURE 4.2. Interaction on a binary tree.

5. Interaction on an n-branch tree

The population is arranged on an n-branch tree, with $n > 2$. So each player has one direct neighbor to the left and n direct neighbors to the right. In terms of a value chain: every chain actor has one supplier but n buyers. One example of a chain actor on an n-branch tree may be a cooperative of coffee bean processors buying beans from one smallholder farmer and supplying to n exporters.

When microentrepreneurs interact on an n-branch tree, there are - from microentrepreneur i's perspective - $n+2$ possible configurations: neither neighbor invests in a safe project; one neighbor invests in a safe project; two neighbors invest in a safe project; ...; n neighbors invest in a safe project; $n+1$ neighbors invest in a safe project. The following conditions need to hold under each possible configuration so that it is a best response for microentrepreneur i to invest in a safe project:

- Given the configuration under which neither neighbor invests in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S - (n+1)c > \mathbb{E}[Y_R] - (n+1)c.$$
- Given the configuration under which one neighbor invests in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S + \Delta_c - nc > \mathbb{E}[Y_R] - (n+1)c,$$
or
$$c + \Delta_c > P.$$
- Given the configuration under which two neighbors invest in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S + 2\Delta_c - (n-1)c > \mathbb{E}[Y_R] - (n+1)c,$$
or
$$2(c + \Delta_c) > P.$$
- ...
- Given the configuration under which all $n+1$ neighbors invest in a safe project, it is a best response for microentrepreneur i to invest in a safe project if
$$Y_S + (n+1)\Delta_c > \mathbb{E}[Y_R] - (n+1)c,$$
or
$$(n+1)(c + \Delta_c) > P.$$

5.1. **Contagion on an n-branch tree.** The contagion threshold for interaction on an n-branch tree is $\xi = \frac{1}{n+1}$. So if $q < \frac{1}{n+1}$, safe investment decisions spread throughout the n-branch tree via best response dynamics.

This requires the risk premium to fulfill the following condition:
$$\frac{\mathbb{E}[Y_R] - c - Y_S}{\Delta_c - c} < \frac{1}{n+1},$$
or
$$\frac{1}{n+1}\Delta_c + \frac{n}{n+1}c > P;$$
which leads us to proposition 5.1.

Proposition 5.1. *Contagion of safe investment decisions occurs throughout the n-branch tree value chain when the risk premium is lower than $\frac{1}{n+1}\Delta_c + \frac{n}{n+1}c$.*

6. Interaction on n-component regions

The population is divided into a number of regions of n microentrepreneurs each. Each microentreprenuer in a region interacts with every other microentrepreneur in that region, as well as with an entrepreneur in each neighboring region.

This structure may describe the situation in which chain actors act independently rather than in cooperatives or unions. This can also be the case when services from one step to the other in the chain need to be highly customized. Take, as in figure 6.1, the local entrepreneurs involved from one end of the value chain to the other, and assume each of the chain functions is carried out by n agents.

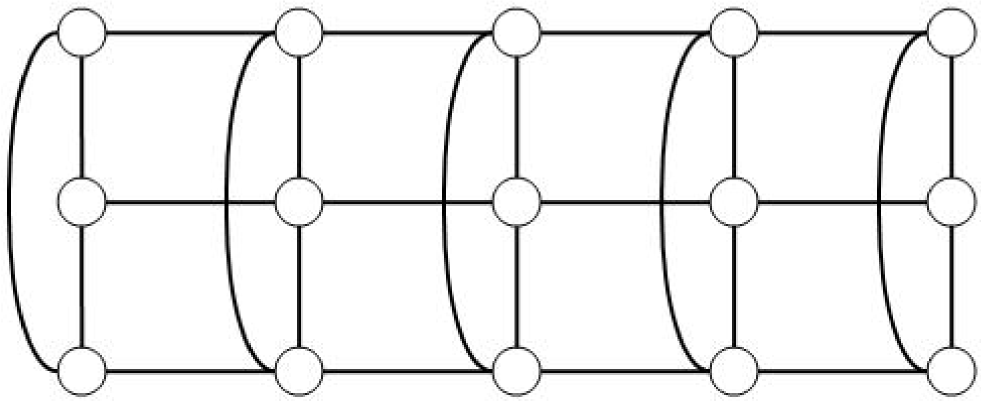

FIGURE 6.1. Interaction on an n-component regional network, with $n = 3$.

When microentrepreneurs interact on n-component regions, there are - from microentrepreneur i's perspective - $n + 2$ possible configurations, just as in the n-branch tree structure analyzed in the previous section. Thus, the conditions that need to hold under each possible configuration so that it is a best response for microentrepreneur i to invest in a safe project are the same as the ones outlined for the n-branch tree.

6.1. **Contagion on n-component regions.** The contagion threshold when interaction takes place in regions of n players is $\xi = \frac{1}{n+1}$, just as for the n-branch tree. So, once again, we get the following condition for the risk premium:

$$\frac{\mathbb{E}\left[Y_R\right] - c - Y_S}{\Delta_c - c} < \frac{1}{n+1},$$

or

$$\frac{1}{n+1}\Delta_c + \frac{n}{n+1}c > P;$$

which leads us to proposition 6.1.

Proposition 6.1. *Contagion of safe investment decisions occurs throughout the n-component regional value chain when the risk premium is lower than $\frac{1}{n+1}\Delta_c + \frac{n}{n+1}c$.*

7. Further discussion

A numerical example may help us summarize the results derived above. Assume all links have current value $c = 100$, and may increase by $\Delta_c = 10$, if business relationships are successful. If chain actors interact along a line, then coordination on the safe project succeeds if

$$P < \frac{1}{2}(100 + 10),$$

i.e. if the risk premium on the risky project is not higher than 55. If, instead, each chain actor interacts with one supplier and two buyers on a binary tree, then coordination on the safe project succeeds if

$$P < \frac{1}{2}(2 * 100 + 10),$$

i.e. if the risk premium on the risky project is not higher than 70. And for the case with three buyers, the risk premium need not be higher than 77.5.

As the number of business neighbors increases, the condition on the risk premium gets looser. Take, for example, interaction on a five-component regional network: the risk premium should then not exceed 85. The upper limit on the risk premium tends to 100 as the number of neighbors tends to infinity.

Proposition 7.1. *As the number of business partners for each chain actor increases, the condition on the risk premium becomes less restrictive and contagion of the good outcome easier to achieve. As n tends to infinity, the upper limit on the risk premium P tends to the value c of a single link.*

Proposition 7.1 confirms the intuition that a chain actor involved in a wider network of partners has more social capital at stake when acting egoistically. She will therefore tend to act in the interest of her network, even when the risk premium is relatively high.

The next proposition summarizes the contagion thresholds for the network structures analyzed in the previous sections.

Proposition 7.2. *The contagion thresholds for different network structures are:*

- *On a line network:*
$$\frac{1}{2}(\Delta_c + c) > P;$$
- *On a binary tree:*
$$\frac{1}{3}(\Delta_c + c) > P;$$
- *On an n-branch tree:*
$$\frac{1}{n+1}\Delta_c + \frac{n}{n+1}c > P;$$
- *On n-component regions:*
$$\frac{1}{n+1}\Delta_c + \frac{n}{n+1}c > P.$$

To see how these results change as the variables of the model change, let us first consider the expected return on the risky and safe projects. As the expected return from the risky project, $\mathbb{E}[Y_R]$, increases (either through an increase of $\pi_{i,R}$ or Y_R), the risk premium increases, making contagion of safe investment choices throughout the network less probable. The same effect occurs when the sure return from the safe project, Y_S, decreases. This effect can be offset by an increase in link values c, by an increase in Δ_c, or both. That is, a higher threat of social sanctions offsets the private benefits an entrepreneur would derive if acting egoistically.

8. Group properties for contagion

Contagion of safe investment choices is easier to achieve throughout networks where the contagion threshold is closer to its upper bound, $\frac{1}{2}$. Morris [Mor00] shows that this condition is fulfilled when a group of nodes satisfies two properties.[12] First of all, there needs to be low neighbor growth, meaning that the number of players who can be reached in k steps grows less than exponentially in k. Low neighbor growth occurs if there is a tendency for players' neighbors' neighbors to be their own neighbors. In social networks, this property is satisfied for groups of closely-related individuals. Since the threat of social sanctions is more credible in small village communities among very close friends and relatives, as in the most typical microfinance settings, it is reasonable to believe that this property holds among microfinance borrowers. Although links between chain actors on an agricultural value chain are of a business nature, family ties cannot be excluded whenever the activities are located in close geographical proximity.

The second group property Morris [Mor00] suggests needs to hold for successful network coordination is that the local interaction system is sufficiently uniform. So there is some number α s.t. for all players a long way from some core group, roughly proportion α of their neighbors are closer to the core group. We may interpret of uniformity in the microfinance context as microentrepreneurs belonging to the same cultural group. Several empirical studies on microfinance lending surveyed by Armendariz and Morduch [AM05] reveal that less diverse groups tend to have higher repayment rates - unless there is a possibility of collusion against the bank. In a trust game experiment conducted in a Peruvian microcredit program, Karlan [Kar07] considers cultural similarity as indicated by language, hair, dress, geographical proximity; he shows that members of groups with higher cultural similarity trust each other more and have higher repayment rates.

9. Conclusions and further research

My aim in this paper has been to apply game-theoretical tools to shed light on the potential for MFIs to supply financial services to a value chain. My focus has been on the extent to which network structure and social cohesion induce chain actors to work towards a socially desirable outcome rather than act solely in their own self-interest.

The main result particularly worthy of note is that a chain actor with a higher number of business links has more social capital at stake and will be less inclined to act egoistically, notwithstanding a relatively high risk premium. This result confirms the intuition that higher social cohesion induces cooperation towards the socially best outcome. When choosing between a safe and a risky investment, a higher threat of social sanctions offsets the private benefits an entrepreneur would derive if acting egoistically.

This paper on interaction games on financial networks is a first step towards a number of related issues I aim to address in future work.[13] First of all, it would be interesting to analyze how the results in this paper change when taking different risk preferences or gradually diminishing / increasing quantities of supplied inputs. Also, how would the results for the n-branch tree change if we inverted the tree, so

---

[12] See Morris [Mor00] for the technical details of these properties.

[13] I thank Paolo Vanini for helpful discussions on possible extensions of this paper.

that each node had multiple suppliers and one buyer (rather than one supplier and multiple buyers)? Another interesting question is whether any specific node in the network is of crucial importance in inducing coordination. Such extensions may help MFIs and traditional banks identify the optimal setting for them to intervene in agricultural value chains.

References

[AB09] F. Allen and A. Babus. Networks in finance. In P. Kleindorfer and Y. Wind, editors, *The Network Challenge: Strategy, Profit, and Risk in an Interlinked World*. 2009.

[AM05] B. Armendariz and J. Morduch. *The Economics of Microfinance*. MIT Press, 2005.

[AMS08] A. Ambrus, M. Mobius, and A. Szeidl. Consumption risk-sharing in social networks. Working paper, Harvard University, 2008.

[CCW07] A. Cassar, L. Crowley, and B. Wydick. The effect of social capital on group loan repayment: Evidence from field experiments. *Economic Journal*, 117:F85–F106, 2007.

[CW09] A. Cassar and B. Wydick. Does social capital matter? Evidence from a five-country group lending experiment. Working paper, University of San Francisco, 2009.

[DJ01] J. Dayton-Johnson. *Social Cohesion and Economic Prosperity*. James Lorimer & Company, 2001.

[FA04] B. Fries and B. Akin. Value chains and their significance for addressing the rural finance challenge. 2004.

[GK08] D. Gale and S. Kariv. Trading in networks: A normal form game experiment. Working paper, New York University, 2008.

[Gla00] M. Gladwell. *The Tipping Point*. Little, Brown & Company, 2000.

[Gra78] M. Granovetter. Threshold models of collective behavior. *American Journal of Sociology*, 83:1420–143, 1978.

[GS03] R. Gomez and E. Santor. Do peer group members outperform individual borrowers? A test of peer group lending using Canadian micro-credit data. Working Paper, Bank of Canada, 2003.

[Kar07] D. Karlan. Social connections and group banking. *Economic Journal*, 117:F52–F84, 2007.

[KMRS09] D. Karlan, M. Mobius, T. Rosenblat, and A. Szeidl. Trust and social collateral. *Quarterly Journal of Economics*, 124:1307–1361, 2009.

[Mor00] S. Morris. Contagion. *Review of Economic Studies*, 67:57–78, 2000.

[Osb03] L. Osberg. *The Economic Implications of Social Cohesion*. University of Toronto Press, 2003.

[Pia08] V. Piana. Working Paper, 2008.

[Qui07] R. Quiros, editor. *Agriculture Value Chain Finance*. FAO, 2007.

[WHK09] B. Wydick, H. Hayes, and S. Kempf. Social networks, neighborhood effects, and credit access: Evidence from rural Guatemala. *World Development*, 2009.

[Wyd99] B. Wydick. Can social cohesion be harnessed to repair market failures? Evidence from group lending in Guatemala. *Economic Journal*, 109:463–475, 1999.

Appendix A. Characterization of the contagion threshold

Morris [Mor00] shows that the contagion threshold corresponds to (i) the smallest p such that every large group contains an infinite $(1-p)$-cohesive subgroup and (ii) the largest p such that it is possible to label players so that, for any player with a sufficiently high label, at least proportion p of his neighbors has a lower label.

Proposition A.1 (Cohesion)**.** *p-cohesion of a group of players describes the self-contained interactions where every microentrepreneur interacts with some other member in the group, rather than with someone outside the group. Cohesion of a group X, $c(X)$, is the smallest p such that every member in X has at least proportion p of her neighbors within X (Morris* [Mor00]*).*

Proposition A.2 (Labeling)**.** *The labeling on a value chain would indicate the position of an entrepreneur in the chain, e.g. a higher/lower label would mean a position closer to the start of the chain. As an example, a coffee bean farmer may have label 1 in a coffee value chain and a coffee shop owner in the city label 10, with other 8 entrepreneurs operating in the middle of the chain.*

The following immediate corollaries of propositions are useful in identifying contagion thresholds in practice.

Corollary A.3 (Upper bound)**.** *If every co-finite group contains an infinite,* $(1-p)$*-cohesive subgroup, then* $\xi \leq p$*.*

Corollary A.4 (Lower bound I)**.** *If there exists a labelling* l *such that* $\alpha_l(k) \geq p$ *for all sufficiently large* k*, then* $\xi \geq p$*.*

An even simpler lower bound is a consequence of corollary A.4.

Corollary A.5 (Lower bound II)**.** *Let* M *be the maximal finite number of neighbors each node can have; then* $\xi \geq \frac{1}{M}$*.*

For completeness of exposition, I reproduce here the derivation by Morris [Mor00] of the contagion threshold for the network structures considered in this paper.

- On a line network:
  Every co-finite group contains an infinite $\frac{1}{2}$-cohesive group of the form $\{x \in \mathbf{Z} : x_1 \geq c\}$. So, by corollary A.3, $\xi \leq \frac{1}{2}$. But $\xi \geq \frac{1}{2}$ by corollary A.5.
- On an n-branch tree:
  Every co-finite group contains an infinite $\frac{n}{n+1}$-cohesive group of the form $\{x \in \bigcup_{n' \geq n} \mathcal{H}_{n'} : x_k = x'_k,\ \text{for each } k = 1, \ldots, n\}$, for some $\hat{x} \in \mathcal{H}_n$. So, by corollary A.3, $\xi \leq \frac{1}{n+1}$. But $\xi \geq \frac{1}{n+1}$ by corollary A.5.
- On an n-component regional network:
  Every co-finite group contains an infinite $\frac{1}{2}$-cohesive group of the form $\{x \in \mathbf{Z} : x_1 \geq c\}$. So, by corollary A.3, $\xi \leq \frac{1}{n+1}$. But $\xi \geq \frac{1}{n+1}$ by corollary A.5.

Zürcher Kantonalbank
E-mail address: `miret.padovani@zkb.ch`